SÉMINAIRES INTERNATIONAUX DE RECHERCHE DE SORBONNE UNIVERSITÉS
UNIVERSITÉ PIERRE ET MARIE CURIE (PARIS 6)

WORKSHOP ON MULTI-LOOP CALCULATIONS (METHODS AND APPLICATIONS)

7-8 JUNE 2017 PARIS FRANCE

UV Divergences and RG Equations in Non-renormalizable Theories

Dmitry Kazakov

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia

Moscow Institute of Physics and Technology, Dolgoprudny, Russia

in collaboration with A. Borlakov, D.Tolkachev and D.Vlasenko

Based on:

Phys.Rev. D95 (2017) no.4, 045006 arXiv:1603.05501 [hep-th]
JHEP 1612 (2016) 154 arXive:1610.05549 [hep-th]

Theory & Harmony
(A glimpse of Paris)

Theory & Harmony
(A glimpse of Paris)

Theory & Harmony
(A glimpse of Paris)

Motivation

Maximal SYM

D=4 N=4
D=6 N=2
D=8 N=1
D=10 N=1

- **Partial or total cancellation of UV divergences (all bubble and triangle diagrams cancel)**
- **First UV divergent diagrams at D=4+6/L**
- **Conformal or dual conformal symmetry**
- **Common structure of the integrands**

Bern, Dixon &Co 10
Drummond, Henn, Korchemsky, Sokatchev 10
Arkani-Hamed 12

Motivation

Maximal SYM

D=4 N=4
D=6 N=2
D=8 N=1
D=10 N=1

- **Partial or total cancell… …ces (all bubble an… …cancel)**
- **First … …ams at D=4+6/L**
  …dual conformal symmetry
  …ion structure of the integrands

All of them can be obtained from 10dim superstring by compactification on a torus

Bern, Dixon &Co 10
Drummond, Henn, Korchemsky, Sokatchev 10
Arkani-Hamed 12

Motivation

Maximal SYM

D=4 N=4
D=6 N=2
D=8 N=1
D=10 N=1

- **Partial or total cancell... ces**
  (all bubble an... cancel)
- **First ... ms at D=4+6/L**
- **...ual conformal symmetry**
- **...ion structure of the integrands**

All of them can be obtained from 10dim superstring by compactification on a torus

Bern, Dixon &Co 10
Drummond, Henn, Korchemsky, Sokatchev 10
Arkani-Hamed 12

D=4 N=8 Supergravity

- **On-shell finite up to 8 loops**
- **Similar to higher dim SYM**

Motivation

Maximal SYM

D=4 N=4
D=6 N=2
D=8 N=1
D=10 N=1

- **Partial or total cancell… …ces (all bubble an… …cancel)**
- **First … …ams at D=4+6/L**
- **…ual conformal symmetry**
- **…ion structure of the integrands**

All of them can be obtained from 10dim superstring by compactification on a torus

Bern, Dixon &Co 10
Drummond, Henn, Korchemsky, Sokatchev 10
Arkani-Hamed 12

D=4 N=8 Supergravity

- **On-shell finite up to 8 loops**
- **Similar to higher dim SYM**

Object: Helicity Amplitudes on mass shell with arbitrary number of legs and loops

The case: Planar limit $N_c \to \infty,\ g^2_{YM} \to 0$ and $g^2_{YM} N_c$ - fixed

The aim: to get all loop (exact) result

Motivation

Maximal SYM

D=4 N=4
D=6 N=2
D=8 N=1
D=10 N=1

- Partial or total cancell... ...ces (all bubble an... ...cancel)
- First ... ...ams at D=4+6/L
- ...ual conformal symmetry
- ...ion structure of the integrands

All of them can be obtained from 10dim superstring by compactification on a torus

Bern, Dixon &Co 10
Drummond, Henn, Korchemsky, Sokatchev 10
Arkani-Hamed 12

D=4 N=8 Supergravity

- On-shell finite up to 8 loops
- Similar to higher dim SYM

Object: Helicity Amplitudes on mass shell with arbitrary number of legs and loops

The case: Planar limit $N_c \to \infty,\ g^2_{YM} \to 0$ and $g^2_{YM} N_c$ - fixed

The aim: to get all loop (exact) result

Study of higher dim SYM gives insight into quantum gravity

UV divergences in all Loops

Spinor-helicity formalism: S-matrix elements

D=4 N=4	**No UV div**	**IR div on shell**
D=6 N=2	**UV div from 3 loops**	**No IR div**
D=8 N=1	**UV div from 1 loop**	**No IR div**
D=10 N=1	**UV div from 1 loop**	**No IR div**

All these theories are non-renormalizable by power counting

The coupling g^2 **has dimension** $[g^2] = \dfrac{1}{M^{D-4}}$

<u>The aim</u>: to get all loop (exact) result for the leading (at least) divs

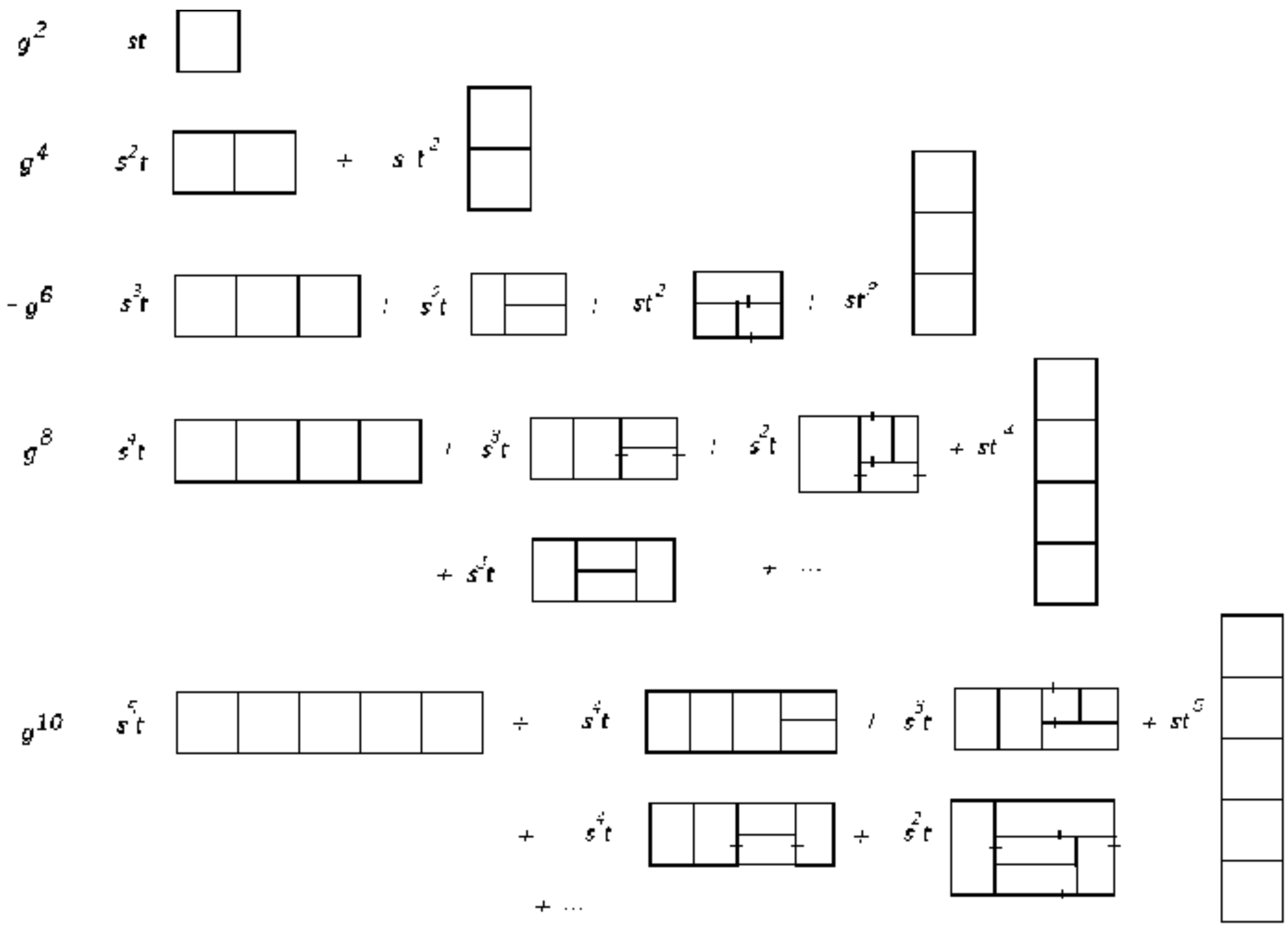
Perturbation Expansion for the 4-point Amplitudes for any D

$$A_4/A_4^{tree}$$

No bubbles
No Triangles

First UV div at L=[6/(D-4)] loops

IR finite



1

2

4

15

60

T. Dennen Yu-yin Huang 10 ,
S.Caron-Huot D.O'Connell 10

Universal expansion for any D in maximal SYM due to Dual conformal invariance

Leading Divergences from Generalized «Renormalization Group»

- **In renormalizable theories the leading divergences can be found from the 1-loop term due to the renormalization group, in particular, for a single coupling theory the coefficient of** $1/\epsilon^n$ **in n loops is**

$$\mathcal{R}'G = \sum_n \frac{a_n^{(n)}}{\epsilon^n} \qquad a_n^{(n)} = (a_1^{(1)})^n$$

Leading Divergences from Generalized «Renormalization Group»

- **In renormalizable theories the leading divergences can be found from the 1-loop term due to the renormalization group, in particular, for a single coupling theory the coefficient of $1/\epsilon^n$ in n loops is**

$$\mathcal{R}'G = \sum_n \frac{a_n^{(n)}}{\epsilon^n} \qquad a_n^{(n)} = (a_1^{(1)})^n$$

- **In non-renormalizable theories the leading divergences can be also found from 1-loop due to locality and R-operation**

$$\mathcal{R}'G = 1 - \sum_\gamma K\mathcal{R}'_\gamma + \sum_{\gamma,\gamma'} K\mathcal{R}'_\gamma K\mathcal{R}'_{\gamma'} - ...,$$

Leading Divergences from Generalized «Renormalization Group»

- **In renormalizable theories the leading divergences can be found from the 1-loop term due to the renormalization group, in particular, for a single coupling theory the coefficient of $1/\epsilon^n$ in n loops is**

$$\mathcal{R}'G = \sum_n \frac{a_n^{(n)}}{\epsilon^n} \qquad a_n^{(n)} = (a_1^{(1)})^n$$

- **In non-renormalizable theories the leading divergences can be also found from 1-loop due to locality and R-operation**

$$\mathcal{R}'G = 1 - \sum_\gamma K\mathcal{R}'_\gamma + \sum_{\gamma,\gamma'} K\mathcal{R}'_\gamma K\mathcal{R}'_{\gamma'} - ...,$$

$$\mathcal{R}'G_n = \frac{A_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^n} + \frac{A_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^n} + ... + \frac{A_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^n}$$
$$+ \frac{B_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^{n-1}} + \frac{B_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^{n-1}} + ... + \frac{B_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^{n-1}}$$
$$+\text{lower order terms}$$

Leading pole

SubLeading pole

$A_1^{(n)},\ B_1^{(n)}$ **1-loop graph**

$B_2^{(n)}$ **2-loop graph**

SubLeading Divergences from Generalized «Renormalization Group»

- **In non-renormalizable theories the leading divergences can be also found from 1-loop due to locality and R-operation**

All terms like $(log\mu^2)^m/\epsilon^k$ **should cancel**

$$A_n^{(n)} = (-1)^{n+1}\frac{A_1^{(n)}}{n},$$

$$B_n^{(n)} = (-1)^n\left(\frac{2}{n}B_2^{(n)} + \frac{n-2}{n}B_1^{(n)}\right)$$

Leading pole from 1 loop diagrams

SubLeading pole from 2 loop diagrams

$$\mathcal{K}\mathcal{R}'G_n = \sum_{k=1}^{n}\left(\frac{A_k^{(n)}}{\epsilon^n} + \frac{B_k^{(n)}}{\epsilon^{n-1}}\right) \equiv \frac{A_n^{(n)'}}{\epsilon^n} + \frac{B_n^{(n)'}}{\epsilon^{n-1}}.$$

$$A_n^{(n)'} = (-1)^{n+1}A_n^{(n)} = \frac{A_1^{(n)}}{n},$$

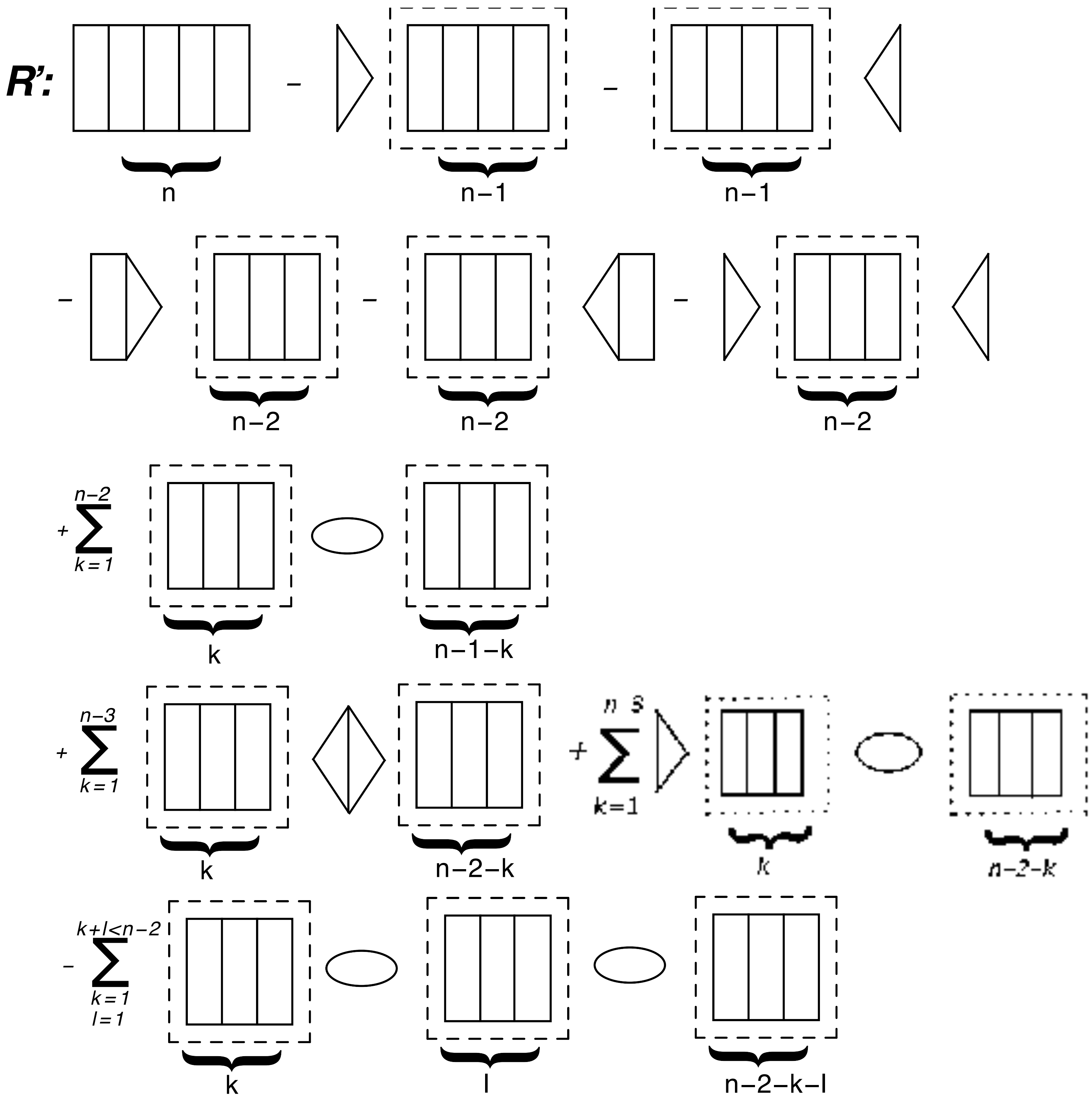
$$B_n^{(n)'} = \left(\frac{2}{n(n-1)}B_2^{(n)} + \frac{2}{n}B_1^{(n)}\right)$$

Just like in renormalizable theories one can deduce the leading, subheading, etc divergences from 1, 2, etc diagrams

R-operation and Recurrence Relation

D=8 N=1

Horizontal boxes



$A_1^{(n)}$ $B_1^{(n)}$

$B_2^{(n)}$

$A_1^{(n)}$ $B_1^{(n)}$

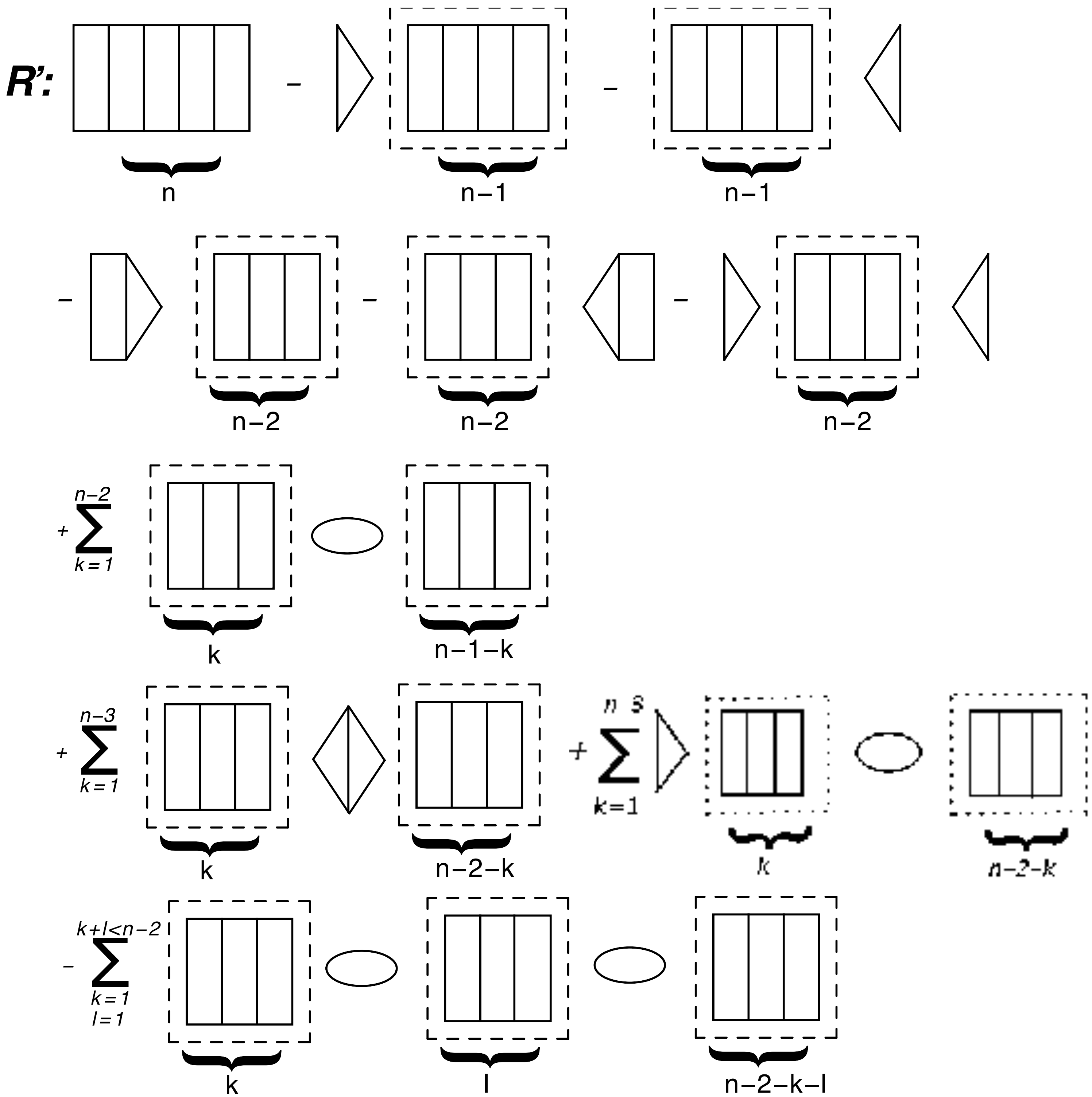
$B_2^{(n)}$

$B_2^{(n)}$

R-operation and Recurrence Relation

D=8 N=1

Horizontal boxes



$A_1^{(n)}$ $B_1^{(n)}$

$B_2^{(n)}$

$A_1^{(n)}$ $B_1^{(n)}$

$B_2^{(n)}$

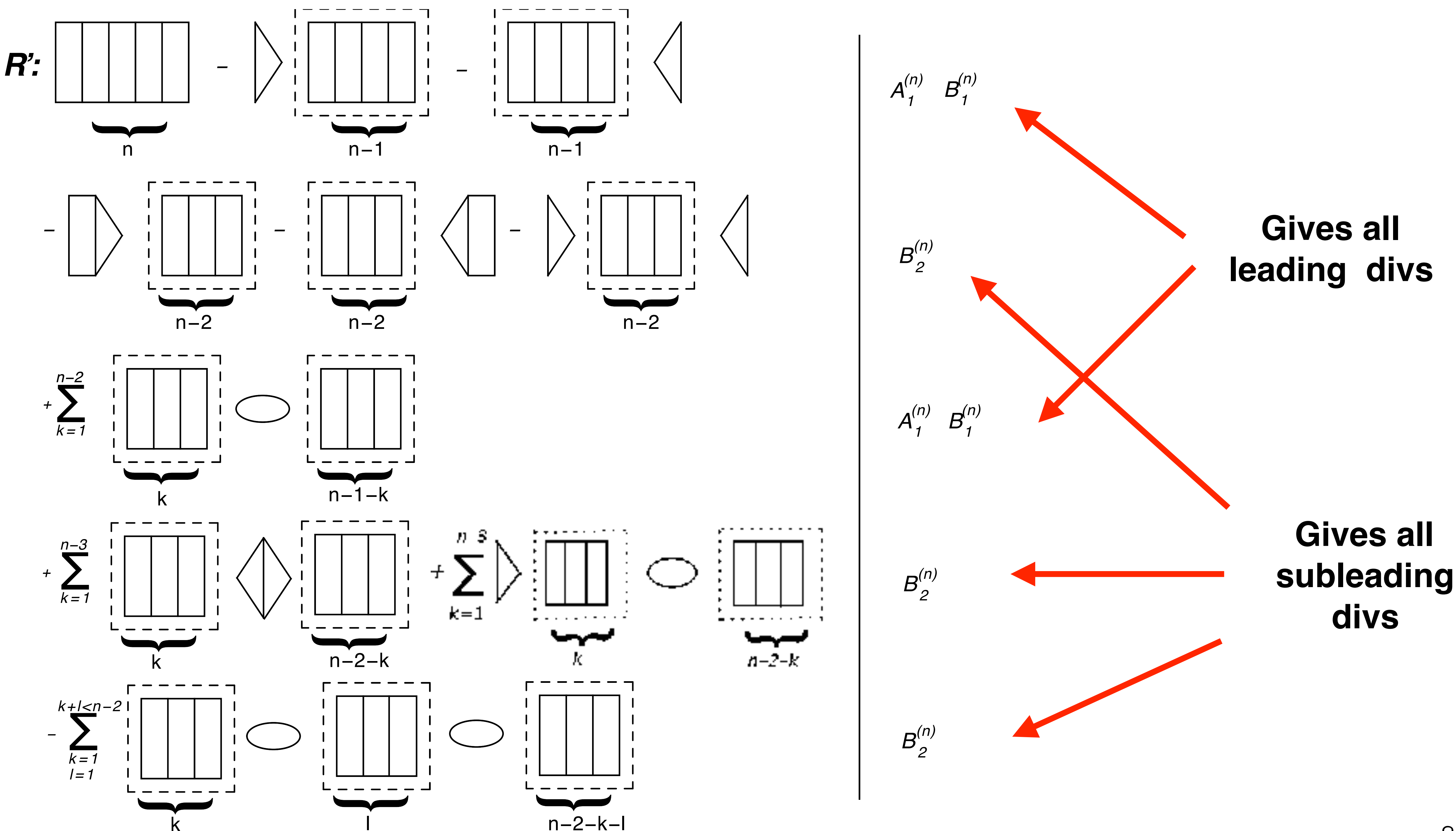
$B_2^{(n)}$

Gives all leading divs

R-operation and Recurrence Relation

D=8 N=1

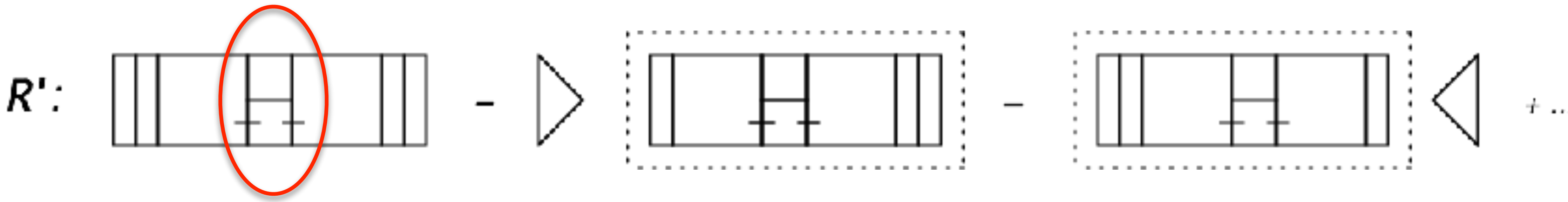
Horizontal boxes



R-operation and Recurrence Relation

D=6 N=2

Horizontal boxes + tennis court



A_n $\qquad$ A_{n-1}

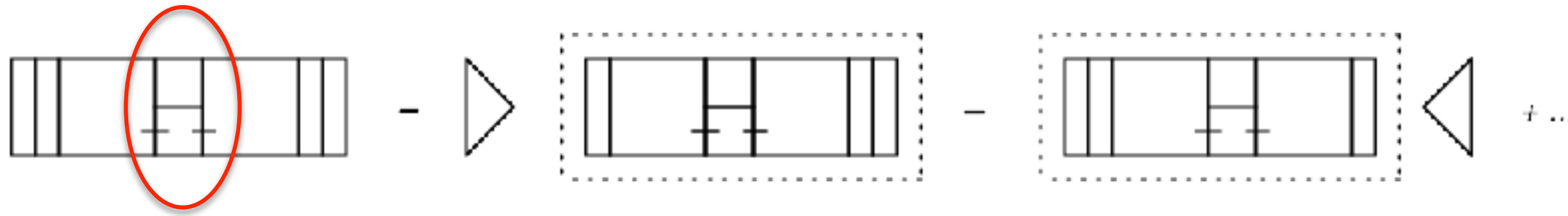
$$nA_n = -A_{n-1} \quad \longrightarrow \quad A_n = (-1)^n \frac{2}{n!} \qquad (-g^2 s)^n$$

R-operation and Recurrence Relation

D=6 N=2

Horizontal boxes + tennis court

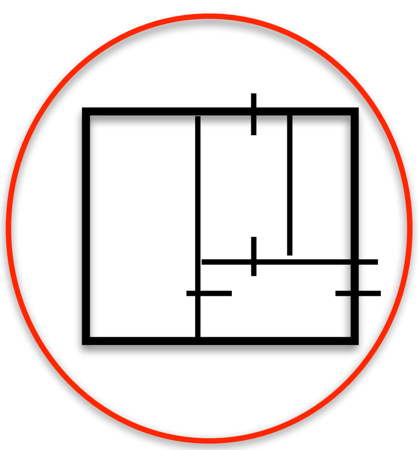
R':



A_n A_{n-1}

$$nA_n = -A_{n-1} \longrightarrow A_n = (-1)^n \frac{2}{n!} \qquad (-g^2 s)^n$$

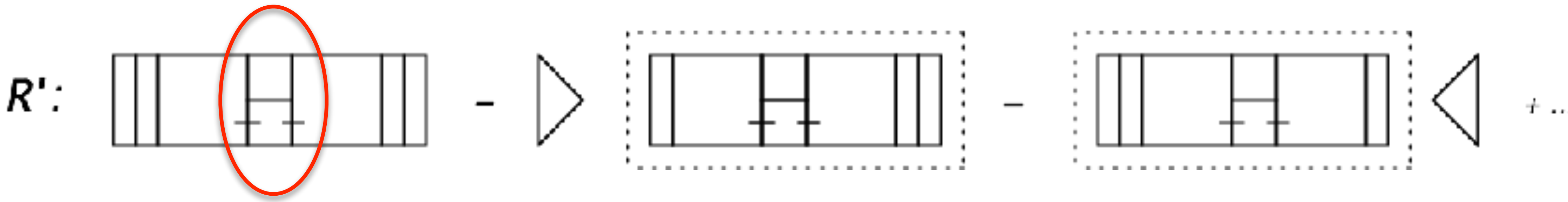
Horizontal boxes + double tennis court



R-operation and Recurrence Relation

D=6 N=2

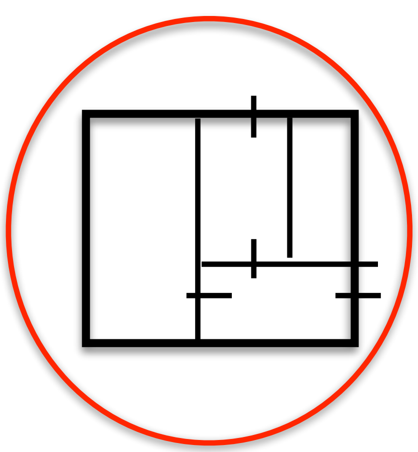
Horizontal boxes + tennis court



A_n $\qquad$ A_{n-1}

$$nA_n = -A_{n-1} \quad \longrightarrow \quad A_n = (-1)^n \frac{2}{n!} \qquad (-g^2 s)^n$$

Horizontal boxes + double tennis court

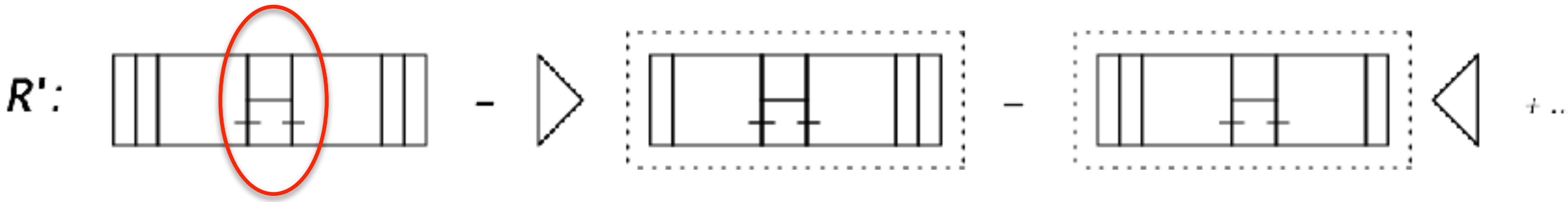


$$nA_n^t = -\frac{1}{3} A_{n-1}^t, \qquad nA_n^s = -A_{n-1}^s + \frac{1}{3} A_{n-1}^t$$

R-operation and Recurrence Relation

D=6 N=2

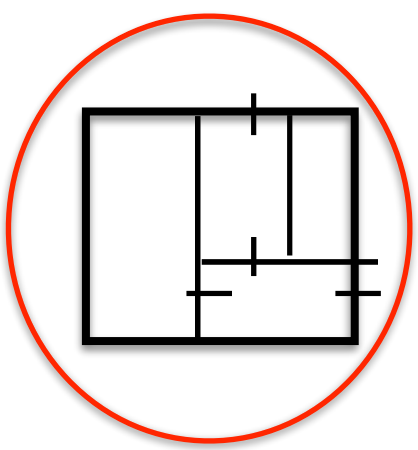
Horizontal boxes + tennis court



A_n $\qquad$ A_{n-1}

$$nA_n = -A_{n-1} \longrightarrow A_n = (-1)^n \frac{2}{n!} \qquad (-g^2 s)^n$$

Horizontal boxes + double tennis court



$$nA_n^t = -\frac{1}{3}A_{n-1}^t, \qquad nA_n^s = -A_{n-1}^s + \frac{1}{3}A_{n-1}^t$$

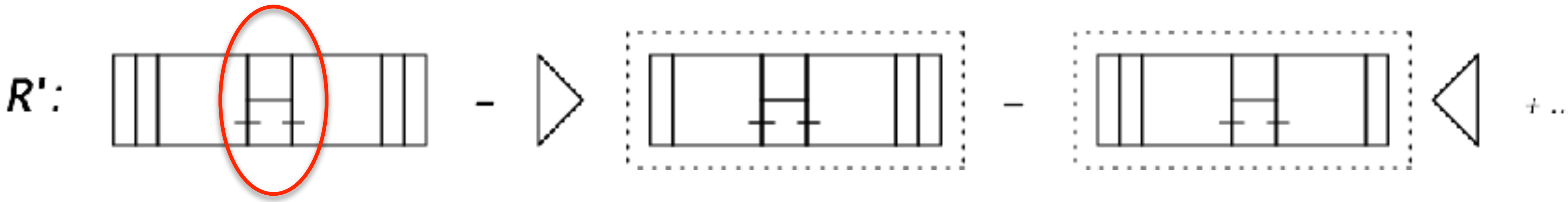
$$\downarrow$$

$$A_n^t = \frac{(-1)^n}{3^{n-3}}\frac{1}{n!}, \qquad A_n^s = \frac{1}{2}\frac{(-1)^n}{3^{n-3}}\frac{1}{n!} - \frac{1}{2}(-1)^n\frac{1}{n!}$$

R-operation and Recurrence Relation

D=6 N=2

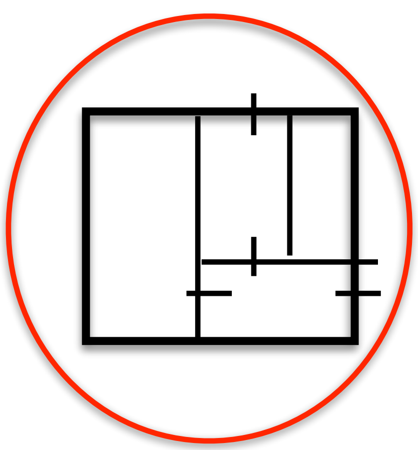
Horizontal boxes + tennis court



A_n $\qquad\qquad\qquad$ A_{n-1}

$$nA_n = -A_{n-1} \longrightarrow A_n = (-1)^n \frac{2}{n!} \qquad (-g^2 s)^n$$

Horizontal boxes + double tennis court



$$nA_n^t = -\frac{1}{3}A_{n-1}^t, \qquad nA_n^s = -A_{n-1}^s + \frac{1}{3}A_{n-1}^t$$

$$\downarrow$$

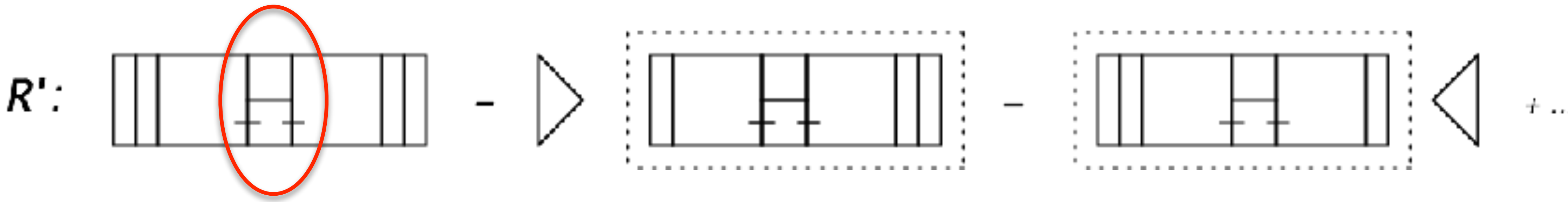
$$A_n^t = \frac{(-1)^n}{3^{n-3}}\frac{1}{n!}, \qquad A_n^s = \frac{1}{2}\frac{(-1)^n}{3^{n-3}}\frac{1}{n!} - \frac{1}{2}(-1)^n\frac{1}{n!}$$

$$(-g^2 s)^{n-1}(-g^2 t)$$

R-operation and Recurrence Relation

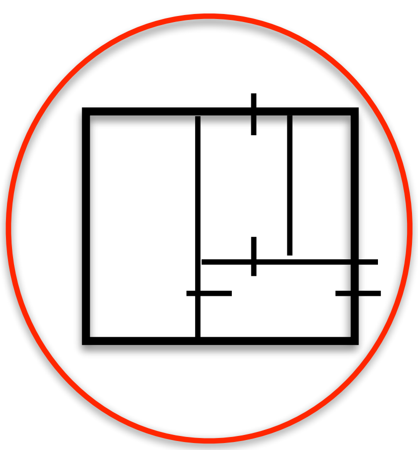
D=6 N=2

Horizontal boxes + tennis court



A_n $\qquad$ A_{n-1}

$$nA_n = -A_{n-1} \quad \longrightarrow \quad A_n = (-1)^n \frac{2}{n!} \qquad (-g^2 s)^n$$



Horizontal boxes + double tennis court

$$nA_n^t = -\frac{1}{3}A_{n-1}^t, \qquad nA_n^s = -A_{n-1}^s + \frac{1}{3}A_{n-1}^t$$

$$\downarrow$$

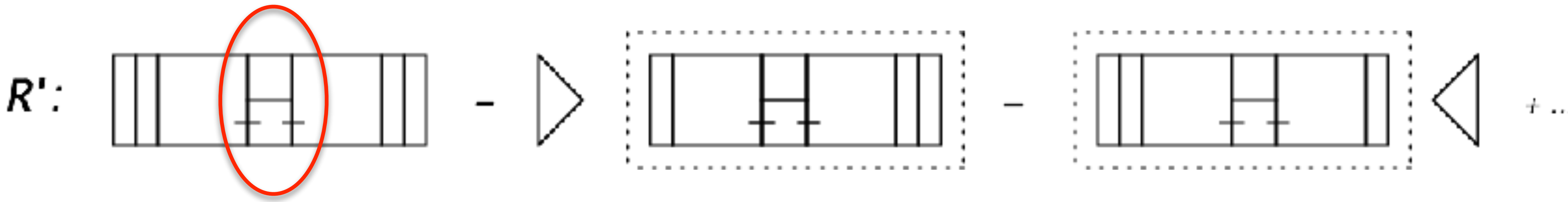
$$A_n^t = \frac{(-1)^n}{3^{n-3}}\frac{1}{n!}, \qquad A_n^s = \frac{1}{2}\frac{(-1)^n}{3^{n-3}}\frac{1}{n!} - \frac{1}{2}(-1)^n\frac{1}{n!}$$

$$(-g^2 s)^{n-1}(-g^2 t) \qquad (-g^2 s)^n$$

R-operation and Recurrence Relation

D=6 N=2

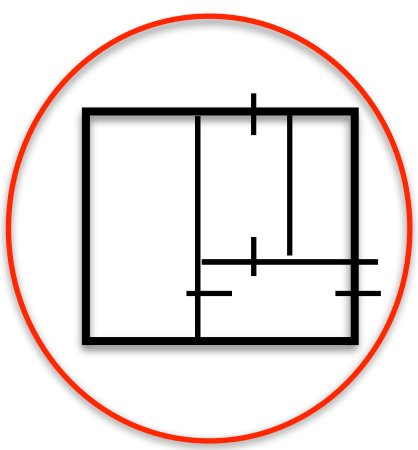
Horizontal boxes + tennis court



A_n A_{n-1}

$$nA_n = -A_{n-1} \quad \longrightarrow \quad A_n = (-1)^n \frac{2}{n!} \qquad (-g^2 s)^n$$

Horizontal boxes + double tennis court



$$nA_n^t = -\frac{1}{3} A_{n-1}^t, \qquad nA_n^s = -A_{n-1}^s + \frac{1}{3} A_{n-1}^t$$

$$A_n^t = \frac{(-1)^n}{3^{n-3}} \frac{1}{n!}, \qquad A_n^s = \frac{1}{2} \frac{(-1)^n}{3^{n-3}} \frac{1}{n!} - \frac{1}{2} (-1)^n \frac{1}{n!}$$

$$(-g^2 s)^{n-1} (-g^2 t) \qquad (-g^2 s)^n$$

- **Similar relations one can get for all other series**
- **All of them have 1/n! behavior**
- **Number of these series group as n!**

Ladder diagrams (leading divs)

D=8 N=1

Horizontal boxes

$$A_n^{(n)} = s^{n-1} A_n$$

$$nA_n = -\frac{2}{4!} A_{n-1} + \frac{2}{5!} \sum_{k=1}^{n-2} A_k A_{n-1-k}, \quad n \geq 3$$

$$A_1 = 1/6$$

1 loop box

Ladder diagrams (leading divs)

D=8 N=1

Horizontal boxes

$$A_n^{(n)} = s^{n-1} A_n$$

$$nA_n = -\frac{2}{4!} A_{n-1} + \frac{2}{5!} \sum_{k=1}^{n-2} A_k A_{n-1-k}, \quad n \geq 3$$

$$A_1 = 1/6$$

1 loop box

Summation

$$\Sigma_m(z) = \sum_{n=m}^{\infty} A_n (-z)^n$$

Ladder diagrams (leading divs)

D=8 N=1 **Horizontal boxes** $A_n^{(n)} = s^{n-1} A_n$

$$nA_n = -\frac{2}{4!}A_{n-1} + \frac{2}{5!}\sum_{k=1}^{n-2} A_k A_{n-1-k}, \quad n \geq 3$$

$A_1 = 1/6$ **1 loop box**

Summation $$\Sigma_m(z) = \sum_{n=m}^{\infty} A_n(-z)^n$$

$$-\frac{d}{dz}\Sigma_3 = -\frac{2}{4!}\Sigma_2 + \frac{2}{5!}\Sigma_1\Sigma_1. \quad \Sigma_3 = \Sigma_1 + A_1 z - A_2 z^2, \quad \Sigma_2 = \Sigma_1 + A_1 z, \quad A_1 = \frac{1}{3!}, \ A_2 = -\frac{1}{3!4!}$$

$\Sigma_A \equiv \Sigma_1$

Diff eqn $$\boxed{\frac{d}{dz}\Sigma_A = -\frac{1}{3!} + \frac{2}{4!}\Sigma_A - \frac{2}{5!}\Sigma_A^2}$$ $z = g^2 s^2/\epsilon$

Ladder diagrams (leading divs)

D=8 N=1 **Horizontal boxes** $A_n^{(n)} = s^{n-1} A_n$

$$nA_n = -\frac{2}{4!}A_{n-1} + \frac{2}{5!}\sum_{k=1}^{n-2} A_k A_{n-1-k}, \quad n \geq 3 \qquad A_1 = 1/6$$

1 loop box

Summation $\Sigma_m(z) = \sum_{n=m}^{\infty} A_n(-z)^n$

$$-\frac{d}{dz}\Sigma_3 = -\frac{2}{4!}\Sigma_2 + \frac{2}{5!}\Sigma_1\Sigma_1. \qquad \Sigma_3 = \Sigma_1 + A_1 z - A_2 z^2, \quad \Sigma_2 = \Sigma_1 + A_1 z, \quad A_1 = \frac{1}{3!}, \; A_2 = -\frac{1}{3!4!}$$

$\Sigma_A \equiv \Sigma_1$

Diff eqn

$$\boxed{\frac{d}{dz}\Sigma_A = -\frac{1}{3!} + \frac{2}{4!}\Sigma_A - \frac{2}{5!}\Sigma_A^2} \qquad z = g^2 s^2/\epsilon$$

$$\boxed{\Sigma_A(z) = -\sqrt{5/3}\frac{4\tan(z/(8\sqrt{15}))}{1-\tan(z/(8\sqrt{15}))\sqrt{5/3}} = \sqrt{10}\frac{\sin(z/(8\sqrt{15}))}{\sin(z/(8\sqrt{15}) - z_0)}}$$

Ladder diagrams (leading divs)

D=8 N=1

Horizontal boxes $A_n^{(n)} = s^{n-1} A_n$

$$nA_n = -\frac{2}{4!}A_{n-1} + \frac{2}{5!}\sum_{k=1}^{n-2} A_k A_{n-1-k}, \quad n \geq 3$$

$A_1 = 1/6$ **1 loop box**

Summation $\Sigma_m(z) = \sum_{n=m}^{\infty} A_n(-z)^n$

$$-\frac{d}{dz}\Sigma_3 = -\frac{2}{4!}\Sigma_2 + \frac{2}{5!}\Sigma_1\Sigma_1. \qquad \Sigma_3 = \Sigma_1 + A_1 z - A_2 z^2, \quad \Sigma_2 = \Sigma_1 + A_1 z, \quad A_1 = \frac{1}{3!}, \; A_2 = -\frac{1}{3!4!}$$

$\Sigma_A \equiv \Sigma_1$

Diff eqn $$\boxed{\frac{d}{dz}\Sigma_A = -\frac{1}{3!} + \frac{2}{4!}\Sigma_A - \frac{2}{5!}\Sigma_A^2}$$ $z = g^2 s^2/\epsilon$

$$\boxed{\Sigma_A(z) = -\sqrt{5/3}\,\frac{4\tan(z/(8\sqrt{15}))}{1 - \tan(z/(8\sqrt{15}))\sqrt{5/3}} = \sqrt{10}\,\frac{\sin(z/(8\sqrt{15}))}{\sin(z/(8\sqrt{15}) - z_0)}}$$

$$\Sigma(z) = -(z/6 + z^2/144 + z^3/2880 + 7z^4/414720 + \dots) \qquad z_0 = \arcsin(\sqrt{3/8})$$

All loop Exact Recurrence Relation

D=6 N=2

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$\boxed{nS_n(s,t) = -2s\int_0^1 dx \int_0^x dy\ (S_{n-1}(s,t') + T_{n-1}(s,t'))}$$

$$n \geq 4$$

$$t' = t(x-y) - sy$$

$$S_3 = -s/3,\ T_3 = -t/3$$

All loop Exact Recurrence Relation

D=6 N=2

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$\boxed{nS_n(s,t) = -2s\int_0^1 dx \int_0^x dy \; (S_{n-1}(s,t') + T_{n-1}(s,t'))}$$

$n \geq 4$

$t' = t(x-y) - sy$

$$S_3 = -s/3, \;\; T_3 = -t/3$$

Summation $$\Sigma_k(s,t,z) = \sum_{n=k}^{\infty} (-z)^n S_n(s,t)$$

All loop Exact Recurrence Relation

D=6 N=2

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$\boxed{nS_n(s,t) = -2s \int_0^1 dx \int_0^x dy \; (S_{n-1}(s,t') + T_{n-1}(s,t'))}$$

$$n \geq 4$$

$$t' = t(x-y) - sy$$

$$S_3 = -s/3, \;\; T_3 = -t/3$$

Summation

$$\Sigma_k(s,t,z) = \sum_{n=k}^{\infty} (-z)^n S_n(s,t)$$

Diff eqn

$$\frac{d}{dz}\Sigma_4(s,t,z) = 2s \int_0^1 dx \int_0^x dy \; (\Sigma_3(s,t',z) + \Sigma_3(t',s,z))|_{t'=xt+yu}$$

All loop Exact Recurrence Relation

D=6 N=2

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$\boxed{nS_n(s,t) = -2s\int_0^1 dx \int_0^x dy\ (S_{n-1}(s,t') + T_{n-1}(s,t'))}$$

$$n \geq 4$$
$$t' = t(x-y) - sy$$

$$S_3 = -s/3,\ T_3 = -t/3$$

Summation $$\Sigma_k(s,t,z) = \sum_{n=k}^{\infty}(-z)^n S_n(s,t)$$

Diff eqn $$\frac{d}{dz}\Sigma_4(s,t,z) = 2s\int_0^1 dx \int_0^x dy\ (\Sigma_3(s,t',z) + \Sigma_3(t',s,z))|_{t'=xt+yu}$$

$$\Sigma_4(s,t,z) = \Sigma_3(s,t,z) + S_3(s,t)z^3 \qquad \Sigma(s,t,z) = z^{-2}\Sigma_3(s,t,z)$$

All loop Exact Recurrence Relation

D=6 N=2

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$\boxed{nS_n(s,t) = -2s\int_0^1 dx \int_0^x dy\ (S_{n-1}(s,t') + T_{n-1}(s,t'))}$$

$$n \geq 4$$

$$t' = t(x-y) - sy$$

$$S_3 = -s/3,\ T_3 = -t/3$$

Summation

$$\Sigma_k(s,t,z) = \sum_{n=k}^{\infty} (-z)^n S_n(s,t)$$

Diff eqn

$$\frac{d}{dz}\Sigma_4(s,t,z) = 2s\int_0^1 dx \int_0^x dy\ (\Sigma_3(s,t',z) + \Sigma_3(t',s,z))|_{t'=xt+yu}$$

$$\Sigma_4(s,t,z) = \Sigma_3(s,t,z) + S_3(s,t)z^3 \qquad \Sigma(s,t,z) = z^{-2}\Sigma_3(s,t,z)$$

$$\boxed{\frac{d}{dz}\Sigma(s,t,z) = s - \frac{2}{z}\Sigma(s,t,z) + 2s\int_0^1 dx \int_0^x dy\ (\Sigma(s,t',z) + \Sigma(t',s,z))|_{t'=xt+yu}}$$

All loop Exact Recurrence Relation

D=8 N=1

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$nS_n(s,t) = -2s^2 \int_0^1 dx \int_0^x dy\; y(1-x)\; \left(S_{n-1}(s,t') + T_{n-1}(s,t')\right)|_{t'=tx+yu}$$

$$+ \quad s^4 \int_0^1 dx\; x^2(1-x)^2 \sum_{k=1}^{n-2} \sum_{p=0}^{2k-2} \frac{1}{p!(p+2)!}\; \frac{d^p}{dt'^p}\left(S_k(s,t') + T_k(s,t')\right) \times$$

$$S_1 = \frac{1}{12},\; T_1 = \frac{1}{12} \qquad \times \frac{d^p}{dt'^p}\left(S_{n-1-k}(s,t') + T_{n-1-k}(s,t')\right)|_{t'=-sx}\; (tsx(1-x))^p$$

All loop Exact Recurrence Relation

D=8 N=1

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$nS_n(s,t) = -2s^2 \int_0^1 dx \int_0^x dy\ y(1-x)\ (S_{n-1}(s,t') + T_{n-1}(s,t'))|_{t'=tx+yu}$$

$$+ \quad s^4 \int_0^1 dx\ x^2(1-x)^2 \sum_{k=1}^{n-2} \sum_{p=0}^{2k-2} \frac{1}{p!(p+2)!}\ \frac{d^p}{dt'^p}(S_k(s,t') + T_k(s,t')) \times$$

$$S_1 = \frac{1}{12},\ T_1 = \frac{1}{12} \qquad \times \frac{d^p}{dt'^p}(S_{n-1-k}(s,t') + T_{n-1-k}(s,t'))|_{t'=-sx}\ (tsx(1-x))^p$$

summation

All loop Exact Recurrence Relation

D=8 N=1

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$nS_n(s,t) = -2s^2 \int_0^1 dx \int_0^x dy \; y(1-x) \; (S_{n-1}(s,t') + T_{n-1}(s,t'))|_{t'=tx+yu}$$

$$+ \quad s^4 \int_0^1 dx \; x^2(1-x)^2 \sum_{k=1}^{n-2} \sum_{p=0}^{2k-2} \frac{1}{p!(p+2)!} \; \frac{d^p}{dt'^p}(S_k(s,t') + T_k(s,t')) \times$$

$$S_1 = \frac{1}{12}, \; T_1 = \frac{1}{12} \qquad \times \frac{d^p}{dt'^p}(S_{n-1-k}(s,t') + T_{n-1-k}(s,t'))|_{t'=-sx} \; (tsx(1-x))^p$$

summation $\Sigma_3(s,t,z) = \Sigma_1(s,t,z) - S_2(s,t)z^2 + S_1(s,t)z, \;\; \Sigma_2(s,t,z) = \Sigma_1(s,t,z) + S_1(s,t)z$

All loop Exact Recurrence Relation

D=8 N=1

s-channel term $S_n(s,t)$ **t-channel term** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact relation for ALL diagrams

$$nS_n(s,t) = -2s^2 \int_0^1 dx \int_0^x dy \; y(1-x) \; (S_{n-1}(s,t') + T_{n-1}(s,t'))|_{t'=tx+yu}$$

$$+ \quad s^4 \int_0^1 dx \; x^2(1-x)^2 \sum_{k=1}^{n-2} \sum_{p=0}^{2k-2} \frac{1}{p!(p+2)!} \; \frac{d^p}{dt'^p}(S_k(s,t') + T_k(s,t')) \times$$

$$S_1 = \frac{1}{12}, \; T_1 = \frac{1}{12} \qquad \times \frac{d^p}{dt'^p}(S_{n-1-k}(s,t') + T_{n-1-k}(s,t'))|_{t'=-sx} \; (tsx(1-x))^p$$

summation $\Sigma_3(s,t,z) = \Sigma_1(s,t,z) - S_2(s,t)z^2 + S_1(s,t)z, \; \Sigma_2(s,t,z) = \Sigma_1(s,t,z) + S_1(s,t)z$

Diff eqn

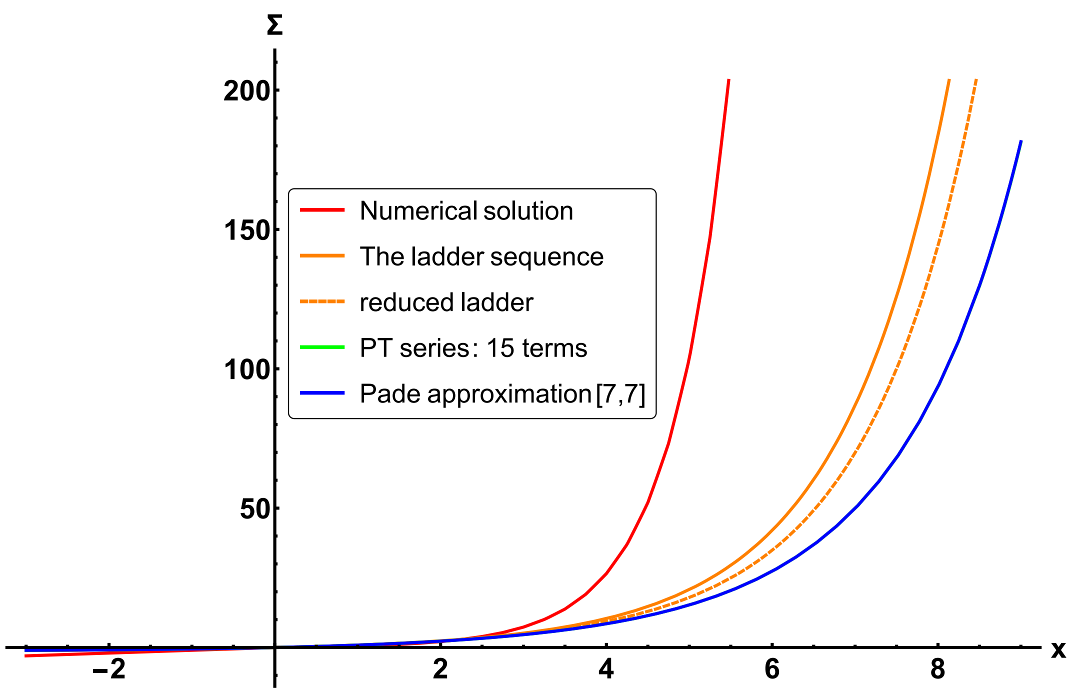
$$\frac{d}{dz}\Sigma(s,t,z) = -\frac{1}{12} + 2s^2 \int_0^1 dx \int_0^x dy \; y(1-x) \; (\Sigma(s,t',z) + \Sigma(t',s,z))|_{t'=tx+yu}$$

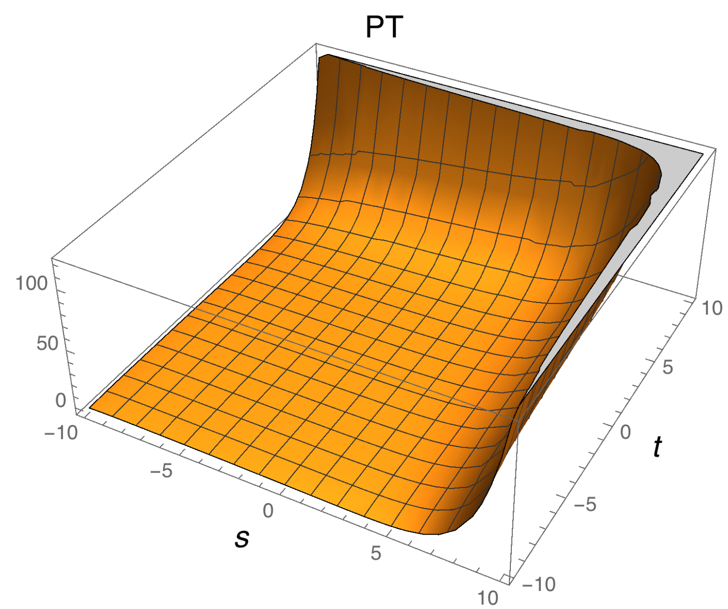
$$-s^4 \int_0^1 dx \; x^2(1-x)^2 \sum_{p=0}^{\infty} \frac{1}{p!(p+2)!} (\frac{d^p}{dt'^p}(\Sigma(s,t',z) + \Sigma(t',s,z))|_{t'=-sx})^2 \; (tsx(1-x))^p.$$

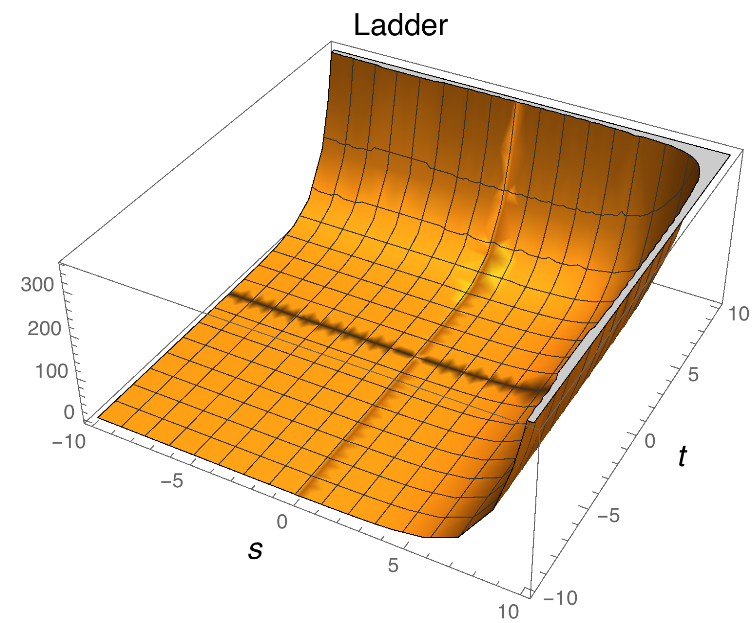
All loop Solution (leading divs)

D=6 N=2

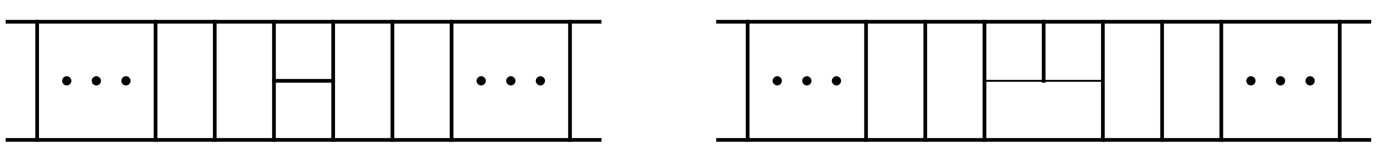
PT (15 terms)





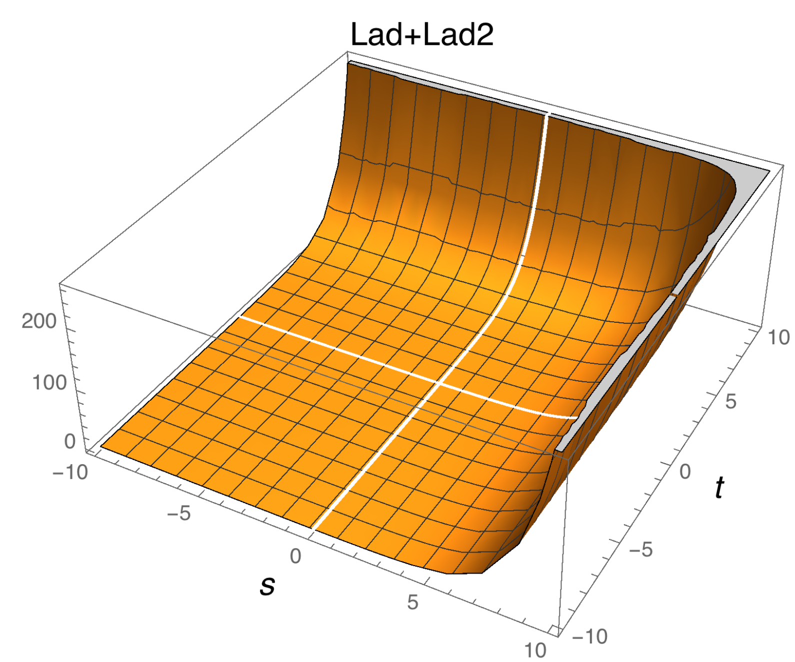


PT and Pade versus ladder for t=s



Ladder

Lddder 2



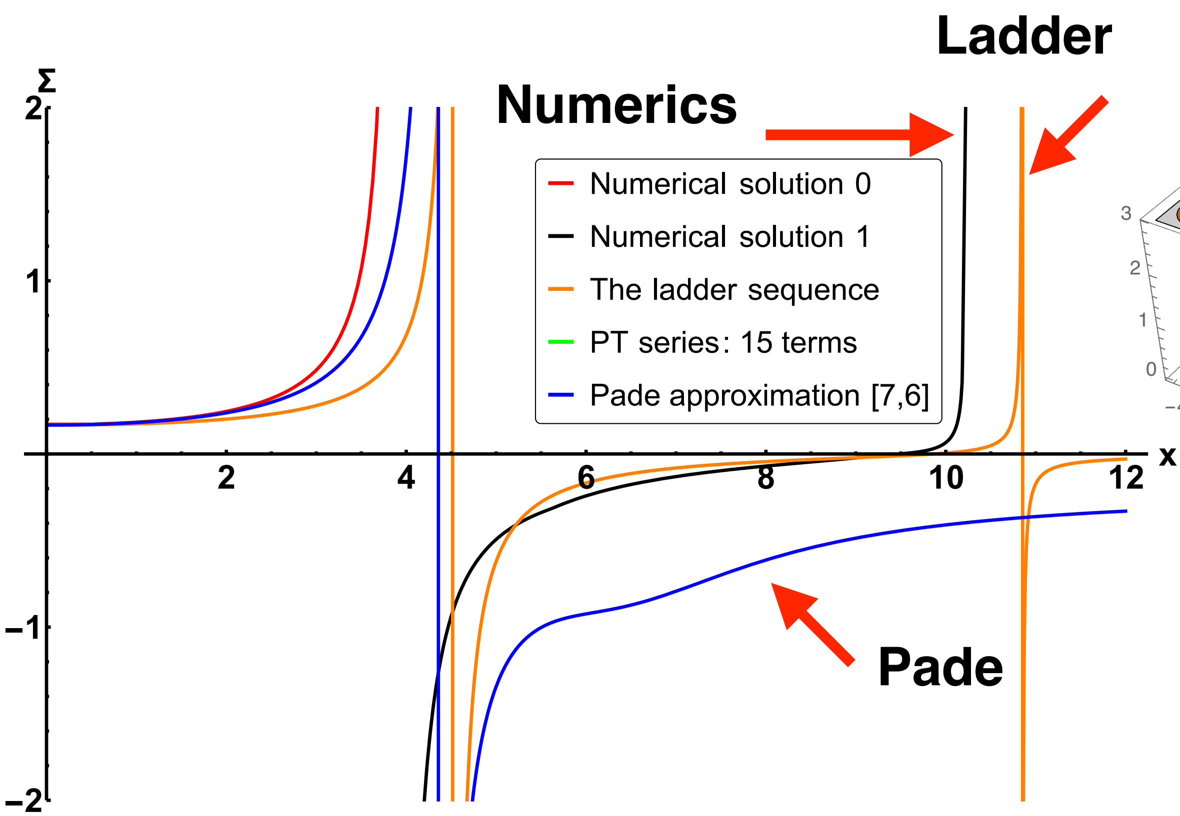
$$\boxed{\Sigma_L(s,z) = \frac{2}{s^2 z^2}\left(e^{sz} - 1 - sz - \frac{s^2 z^2}{2}\right)}$$

$$z = \frac{g^2}{\epsilon}$$

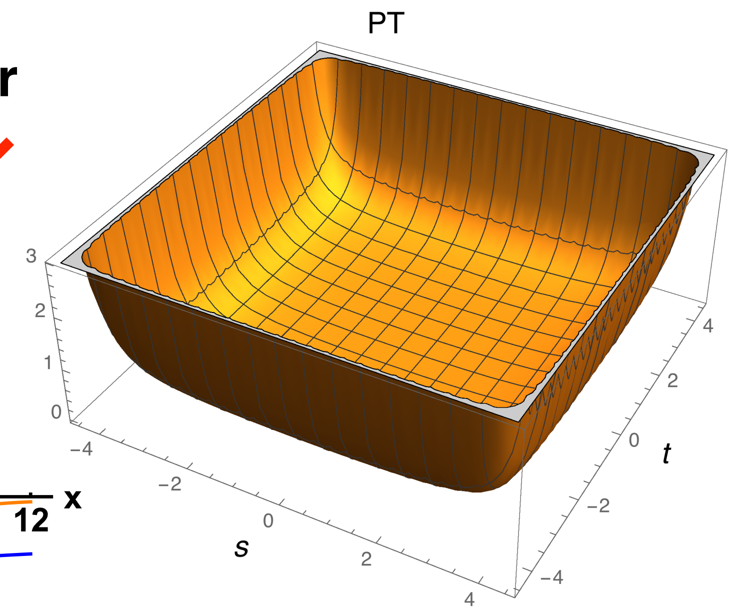
Numerical solution of the full equation is close to the ladder approx

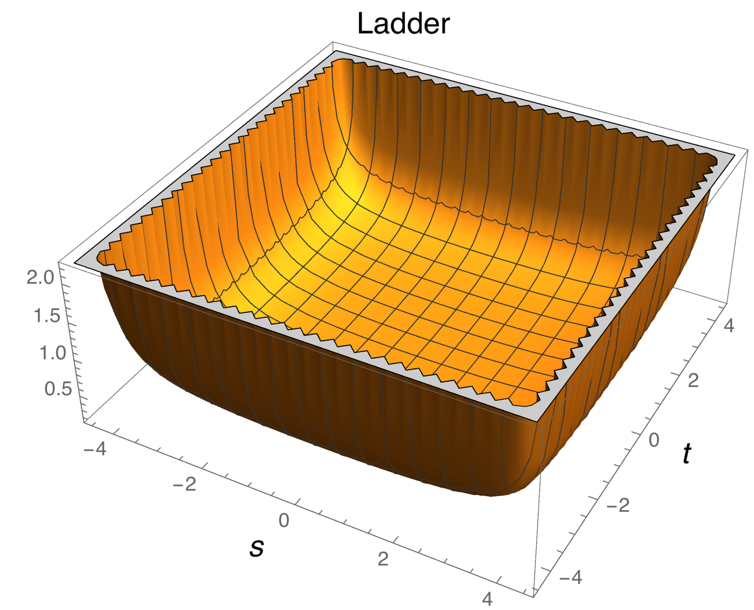
All loop Solution (leading divs)

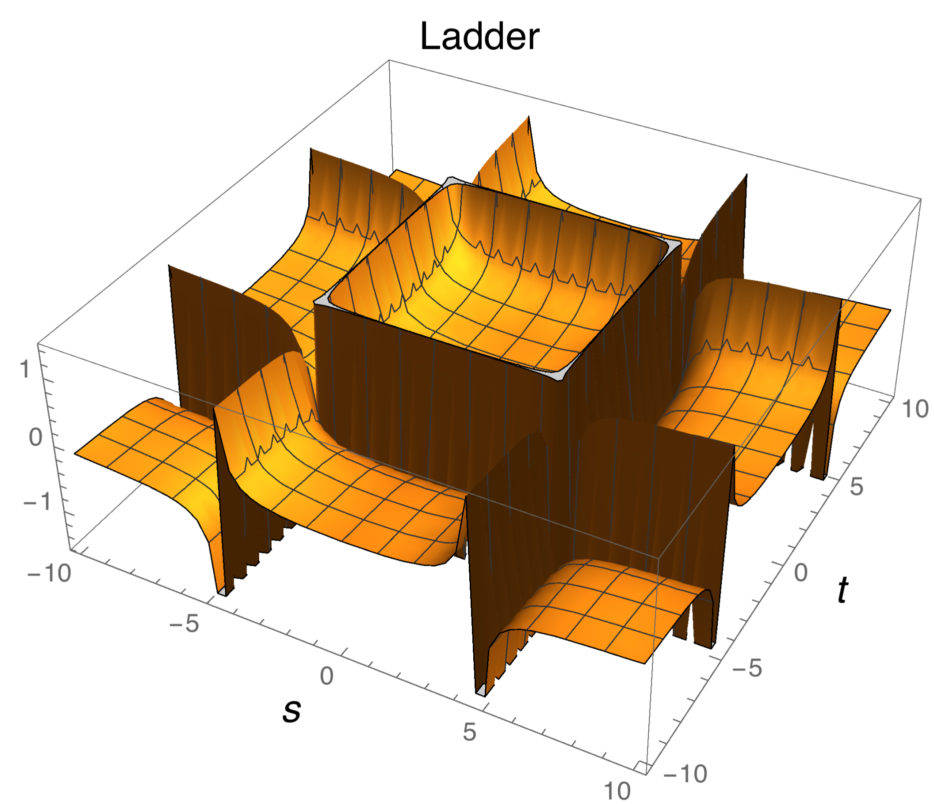
D=8 N=1

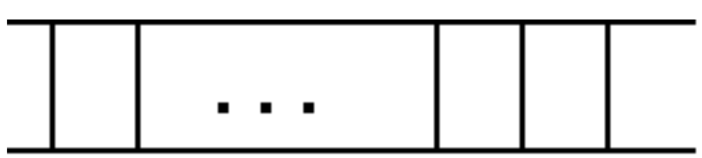


PT and Pade versus ladder for t=s









$$z = \frac{g^2}{\epsilon}$$

$$\boxed{\Sigma_L(s,z) = -\sqrt{5/3}\,\frac{4\tan(zs^2/(8\sqrt{15}))}{1-\tan(zs^2/(8\sqrt{15}))\sqrt{5/3}}}$$

Ladder diagrams (subleading divs)

$$A_n^{(n)} = s^{n-1} A_n, A_n^{(n)'} = s^{n-1} A'_n,$$

$$B_n^{(n)} = s^{n-1} B_{sn} + s^{n-2} t B_{tn},\ B_n^{(n)'} = s^{n-1} B'_{sn} + s^{n-2} t B'_{tn}$$

Ladder diagrams (subleading divs)

$$A_n^{(n)} = s^{n-1} A_n, A_n^{(n)'} = s^{n-1} A'_n,$$

$$B_n^{(n)} = s^{n-1} B_{sn} + s^{n-2} t B_{tn},\ B_n^{(n)'} = s^{n-1} B'_{sn} + s^{n-2} t B'_{tn}$$

$$B'_{tn} = -\frac{2}{n(n-1)} B'_{tn-2} \frac{2}{5!5!} + \frac{2}{n} B'_{tn-1} \frac{2}{5!}$$

Ladder diagrams (subleading divs)

$$A_n^{(n)} = s^{n-1}A_n, A_n^{(n)'} = s^{n-1}A'_n,$$

$$B_n^{(n)} = s^{n-1}B_{sn} + s^{n-2}tB_{tn},\ B_n^{(n)'} = s^{n-1}B'_{sn} + s^{n-2}tB'_{tn}$$

$$B'_{tn} = -\frac{2}{n(n-1)}B'_{tn-2}\frac{2}{5!5!} + \frac{2}{n}B'_{tn-1}\frac{2}{5!} \qquad B_{tn} = (-1)^n\left[-\frac{2}{n}B'_{tn-2}\frac{2}{5!5!} + \frac{n-2}{n}B'_{tn-1}\frac{2}{5!}\right]$$

Ladder diagrams (subleading divs)

$$A_n^{(n)} = s^{n-1}A_n, A_n^{(n)'} = s^{n-1}A'_n,$$

$$B_n^{(n)} = s^{n-1}B_{sn} + s^{n-2}tB_{tn}, \; B_n^{(n)'} = s^{n-1}B'_{sn} + s^{n-2}tB'_{tn}$$

$$B'_{tn} = -\frac{2}{n(n-1)}B'_{tn-2}\frac{2}{5!5!} + \frac{2}{n}B'_{tn-1}\frac{2}{5!} \qquad B_{tn} = (-1)^n\left[-\frac{2}{n}B'_{tn-2}\frac{2}{5!5!} + \frac{n-2}{n}B'_{tn-1}\frac{2}{5!}\right]$$

$$\begin{aligned}
B'_{sn} = \frac{2}{n(n-1)}\Bigg[&-A'_{n-2}\frac{2321}{5!5!2} - B'_{sn-2}\frac{18}{4!5!} + B'_{tn-2}\frac{44}{5!5!} \\
&-\sum_{k=1}^{n-3}A'_kA'_{n-2-k}\frac{2}{5!3}\frac{47}{15} - \sum_{k=1}^{n-3}A'_kB'_{sn-2-k}\frac{2}{5!12} + \sum_{k=1}^{n-3}A'_kB'_{tn-2-k}\frac{2}{4!5!12} \\
&-\sum_{k,l=1}^{n-k+l<n-2}A'_kA'_lA'_{n-2-k-l}\frac{8}{5!5!}\frac{46}{15} - \sum_{k,l=1}^{n-k+l<n-2}A'_kA'_lB'_{sn-2-k-l}\frac{12}{5!5!} \\
&+\sum_{k,l=1}^{n-k+l<n-2}A'_kA'_lB'_{tn-2-k-l}\frac{4}{5!5!} + \sum_{k,l=1}^{n-k+l<n-2}B'_kA'_lA'_{sn-2-k-l}\frac{2}{5!5!}\Bigg] \\
&+\frac{2}{n}\Bigg[A'_{n-1}\frac{19}{34!} + B'_{sn-1}\frac{2}{4!} - B'_{tn-1}\frac{4}{5!} \\
&+\sum_{k=1}^{n-2}A'_kA'_{n-2-k}\frac{2}{5!}\frac{46}{15} + \sum_{k=1}^{n-2}A'_kB'_{sn-1-k}\frac{4}{5!} - \sum_{k=1}^{n-2}A'_kB'_{tn-1-k}\frac{2}{5!}\Bigg]
\end{aligned}$$

Sum of Ladder diagrams (subleading divs)

summation

$$\Sigma'_{tB} = \sum_{n=2}^{\infty} z^n B'_{tn}$$

$$z = \frac{g^2 s^2}{\epsilon}$$

Sum of Ladder diagrams (subleading divs)

summation

$$\Sigma'_{tB} = \sum_{n=2}^{\infty} z^n B'_{tn}$$

Diff eqn

$$\frac{d^2\Sigma'_{tB}(z)}{dz^2} - \frac{1}{30}\frac{d\Sigma'_{tB}(z)}{dz} + \frac{\Sigma'_{tB}(z)}{720} = -\frac{1}{432}$$

$$z = \frac{g^2 s^2}{\epsilon}$$

Sum of Ladder diagrams (subleading divs)

summation

$$\Sigma'_{tB} = \sum_{n=2}^{\infty} z^n B'_{tn}$$

Diff eqn

$$\frac{d^2\Sigma'_{tB}(z)}{dz^2} - \frac{1}{30}\frac{d\Sigma'_{tB}(z)}{dz} + \frac{\Sigma'_{tB}(z)}{720} = -\frac{1}{432}$$

$$z = \frac{g^2 s^2}{\epsilon}$$

solution

$$\boxed{\Sigma'_{tB}(z) = \frac{5}{6}\left[e^{z/60}(2\cos(z/30) - \sin(z/30)) - 2\right]}$$

Sum of Ladder diagrams (subleading divs)

summation

$$\Sigma'_{tB} = \sum_{n=2}^{\infty} z^n B'_{tn}$$

Diff eqn

$$\frac{d^2\Sigma'_{tB}(z)}{dz^2} - \frac{1}{30}\frac{d\Sigma'_{tB}(z)}{dz} + \frac{\Sigma'_{tB}(z)}{720} = -\frac{1}{432}$$

$$z = \frac{g^2 s^2}{\epsilon}$$

solution

$$\boxed{\Sigma'_{tB}(z) = \frac{5}{6}\left[e^{z/60}(2\cos(z/30) - \sin(z/30)) - 2\right]}$$

$$\Sigma_{tB} = \sum_{n=2}^{\infty}(-z)^n B_{tn}$$

Sum of Ladder diagrams (subleading divs)

summation

$$\Sigma'_{tB} = \sum_{n=2}^{\infty} z^n B'_{tn}$$

Diff eqn

$$\frac{d^2\Sigma'_{tB}(z)}{dz^2} - \frac{1}{30}\frac{d\Sigma'_{tB}(z)}{dz} + \frac{\Sigma'_{tB}(z)}{720} = -\frac{1}{432}$$

$$z = \frac{g^2 s^2}{\epsilon}$$

solution

$$\boxed{\Sigma'_{tB}(z) = \frac{5}{6}\left[e^{z/60}(2\cos(z/30) - \sin(z/30)) - 2\right]}$$

$$\Sigma_{tB} = \sum_{n=2}^{\infty}(-z)^n B_{tn}$$

Diff eqn

$$\frac{d\Sigma_{tB}(z)}{dz} = \frac{1}{60} z \frac{d\Sigma'_{tB}(z)}{dz} - \frac{\Sigma'_{tB}(z)}{60} - z\frac{\Sigma'_{tB}(z)}{720} - \frac{z}{432}$$

Sum of Ladder diagrams (subleading divs)

summation

$$\Sigma'_{tB} = \sum_{n=2}^{\infty} z^n B'_{tn}$$

Diff eqn

$$\frac{d^2\Sigma'_{tB}(z)}{dz^2} - \frac{1}{30}\frac{d\Sigma'_{tB}(z)}{dz} + \frac{\Sigma'_{tB}(z)}{720} = -\frac{1}{432}$$

$$z = \frac{g^2 s^2}{\epsilon}$$

solution

$$\boxed{\Sigma'_{tB}(z) = \frac{5}{6}\left[e^{z/60}(2\cos(z/30) - \sin(z/30)) - 2\right]}$$

$$\Sigma_{tB} = \sum_{n=2}^{\infty}(-z)^n B_{tn}$$

Diff eqn

$$\frac{d\Sigma_{tB}(z)}{dz} = \frac{1}{60} z \frac{d\Sigma'_{tB}(z)}{dz} - \frac{\Sigma'_{tB}(z)}{60} - z\frac{\Sigma'_{tB}(z)}{720} - \frac{z}{432}$$

solution

$$\boxed{\Sigma_{tB} = -\frac{1}{36}\left[60 + z + e^{z/60}(-(60+z)\cos(z/30) - 2(-15+z)\sin(z/30))\right]}$$

Sum of Ladder diagrams (subleading divs)

$$\Sigma'_{sB} = \sum_{n=2}^{\infty} z^n B'_{sn}$$

$$\boxed{\frac{d^2\Sigma'_{sB}(z)}{dz^2} + f_1(z)\frac{d\Sigma'_{sB}(z)}{dz} + f_2(z)\Sigma'_{sB}(z) = f_3(z)}$$

Diff eqn

$$f_1(z) = -\frac{1}{6} + \frac{\Sigma_A}{15},$$

$$f_2(z) = \frac{1}{80} - \frac{\Sigma_A}{360} + \frac{\Sigma_A^2}{600} + \frac{1}{15}\frac{d\Sigma_A}{dz},$$

$$f_3(z) = \frac{2321}{5!5!2}\Sigma_A + \frac{11}{1800}\Sigma'_{tB} - \frac{47}{5!45}\Sigma_A^2 - \frac{1}{5!72}\Sigma_A\Sigma'_{tB} + \frac{23}{6750}\Sigma_A^3 + \frac{1}{1200}\Sigma_A^2\Sigma'_{tB}$$
$$-\frac{19}{36}\frac{d\Sigma_A}{dz} - \frac{1}{15}\frac{d\Sigma'_{tB}}{dz} + \frac{23}{225}\frac{d\Sigma_A^2}{dz} + \frac{1}{30}\frac{d(\Sigma_A\Sigma'_{tB})}{dz} - \frac{3}{32}$$

Sum of Ladder diagrams (subleading divs)

$$\Sigma'_{sB} = \sum_{n=2}^{\infty} z^n B'_{sn}$$

$$\boxed{\frac{d^2\Sigma'_{sB}(z)}{dz^2} + f_1(z)\frac{d\Sigma'_{sB}(z)}{dz} + f_2(z)\Sigma'_{sB}(z) = f_3(z)}$$

Diff eqn

$$f_1(z) = -\frac{1}{6} + \frac{\Sigma_A}{15},$$

$$f_2(z) = \frac{1}{80} - \frac{\Sigma_A}{360} + \frac{\Sigma_A^2}{600} + \frac{1}{15}\frac{d\Sigma_A}{dz},$$

$$f_3(z) = \frac{2321}{5!5!2}\Sigma_A + \frac{11}{1800}\Sigma'_{tB} - \frac{47}{5!45}\Sigma_A^2 - \frac{1}{5!72}\Sigma_A\Sigma'_{tB} + \frac{23}{6750}\Sigma_A^3 + \frac{1}{1200}\Sigma_A^2\Sigma'_{tB} - \frac{19}{36}\frac{d\Sigma_A}{dz} - \frac{1}{15}\frac{d\Sigma'_{tB}}{dz} + \frac{23}{225}\frac{d\Sigma_A^2}{dz} + \frac{1}{30}\frac{d(\Sigma_A\Sigma'_{tB})}{dz} - \frac{3}{32}$$

Solution to Diff eqn

smooth monotonic function

$$\Sigma'_{sB}(z) = \frac{d\Sigma_A}{dz}u(z) \qquad u(z) = \int_0^z dy \int_0^y dx \frac{f_3(x)}{d\Sigma_A(x)/dx}$$

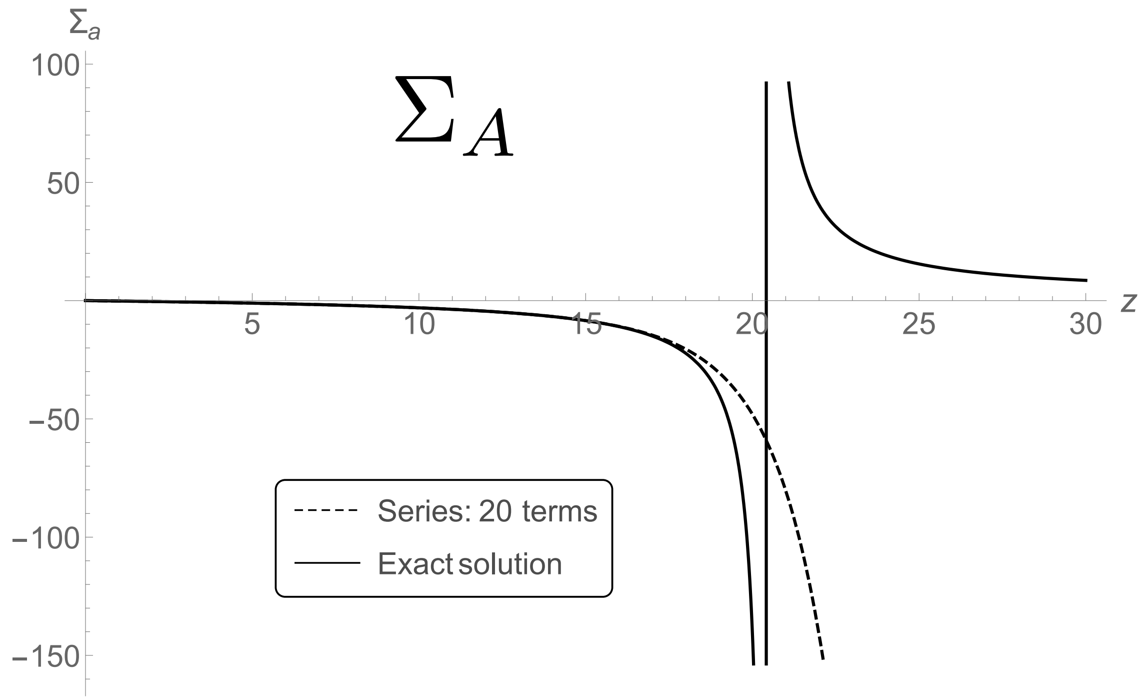
Sum of the Ladder diagrams

solutions

Sum of the Ladder diagrams

solutions

Leading divs

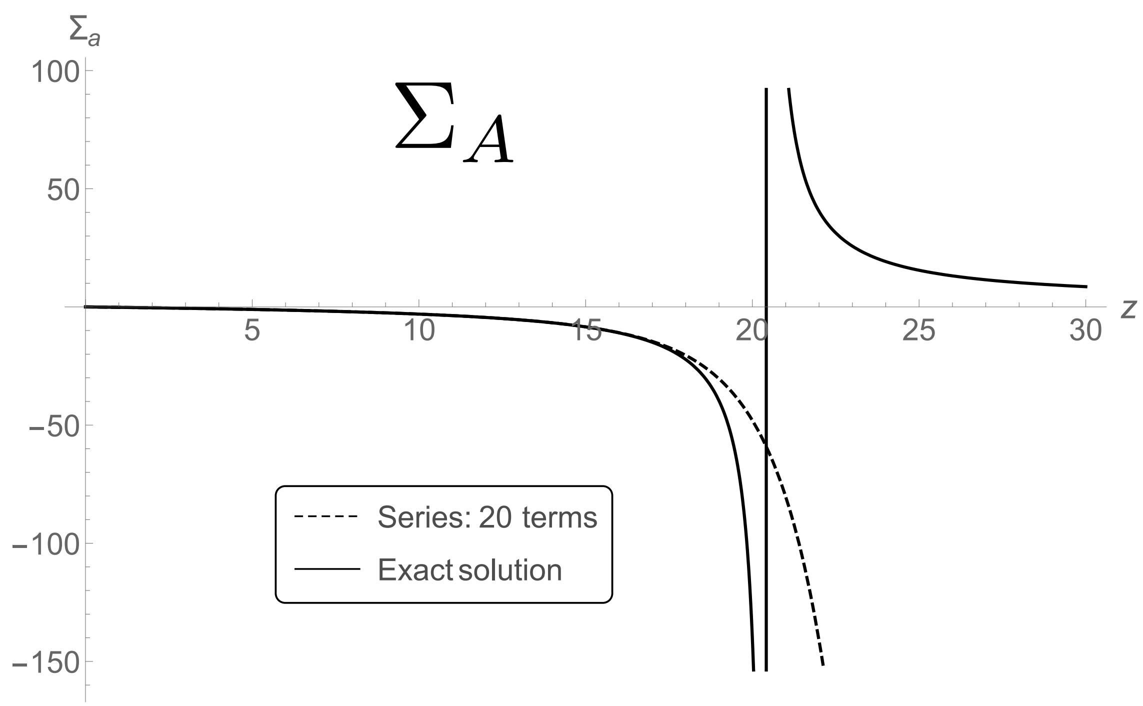


Infinite number of poles

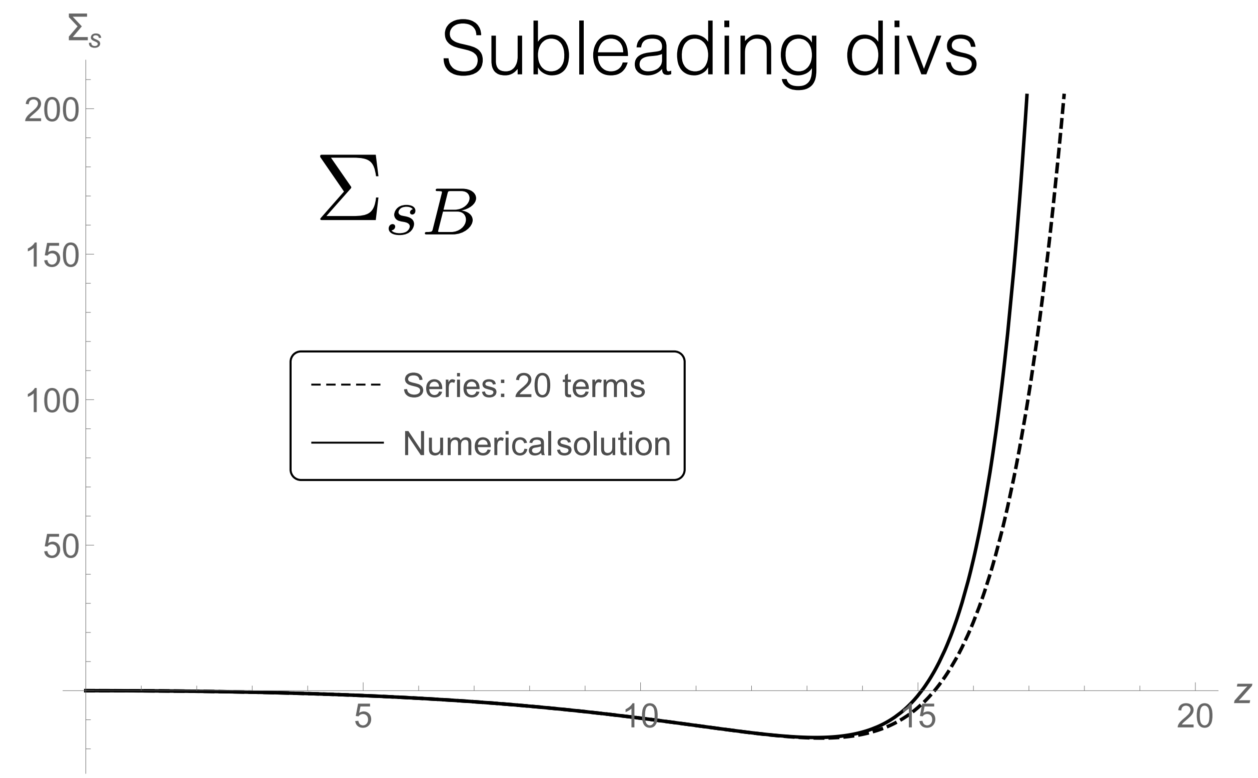
Sum of the Ladder diagrams

solutions

Leading divs



Subleading divs

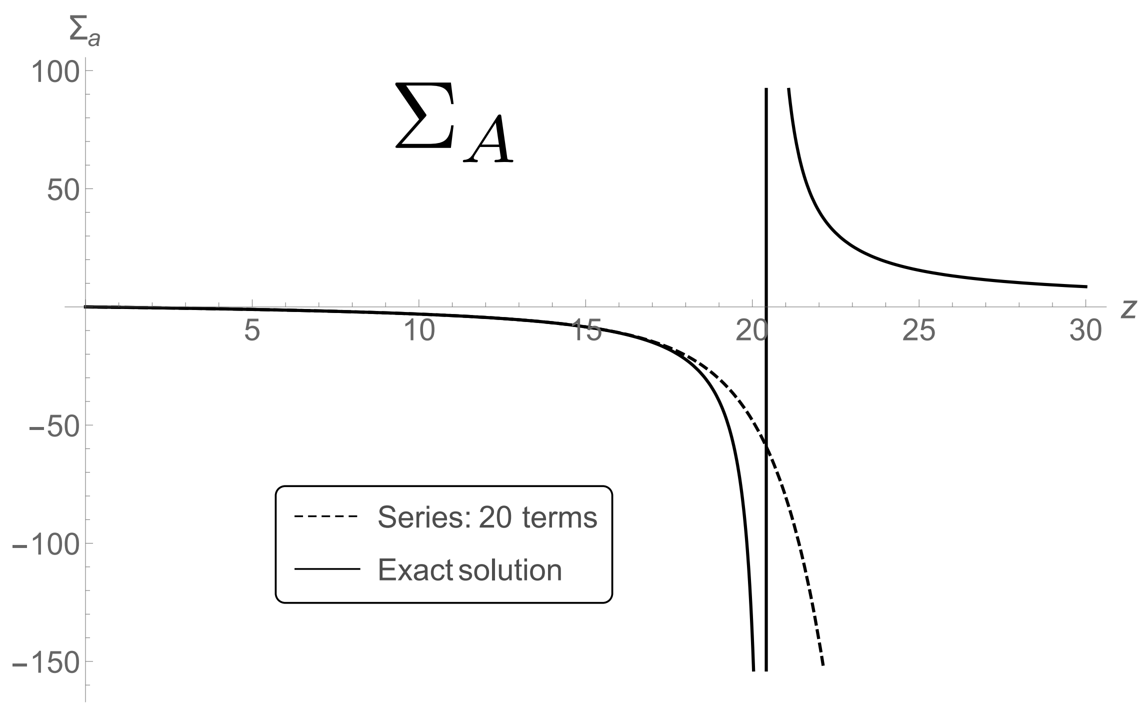


Infinite number of poles

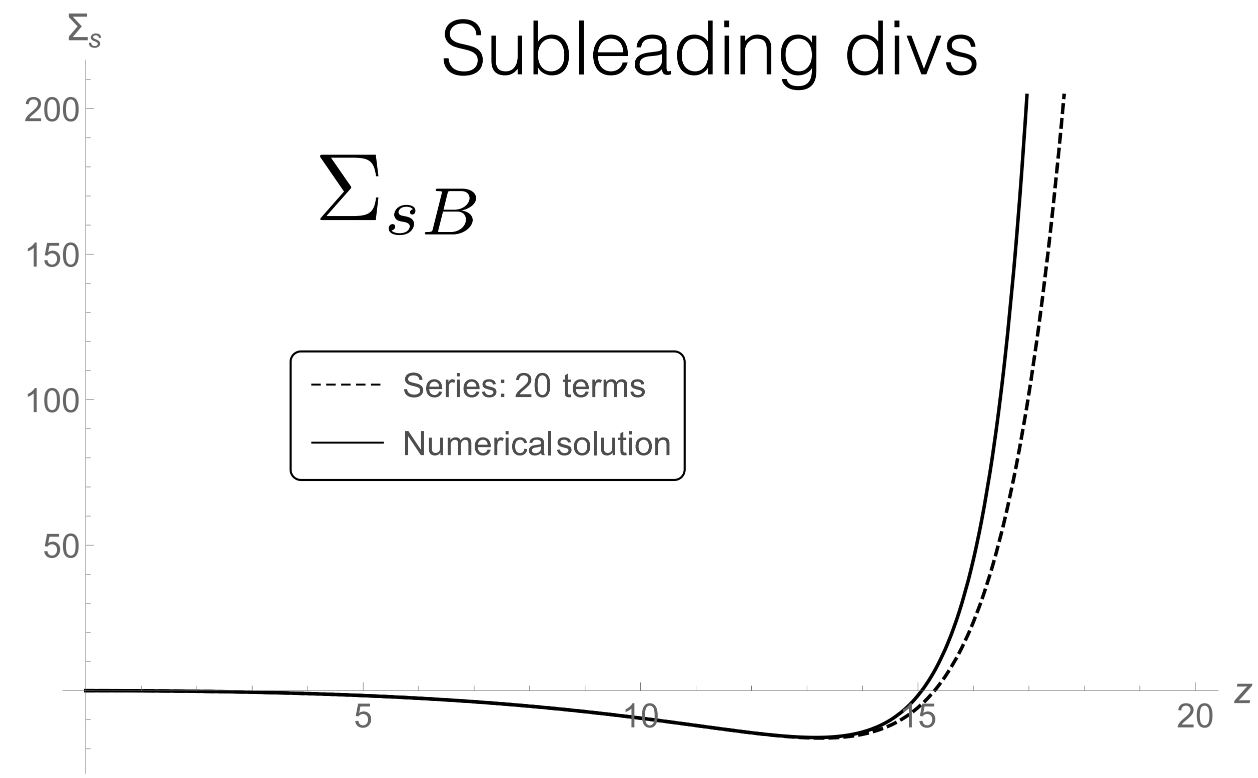
Sum of the Ladder diagrams

solutions

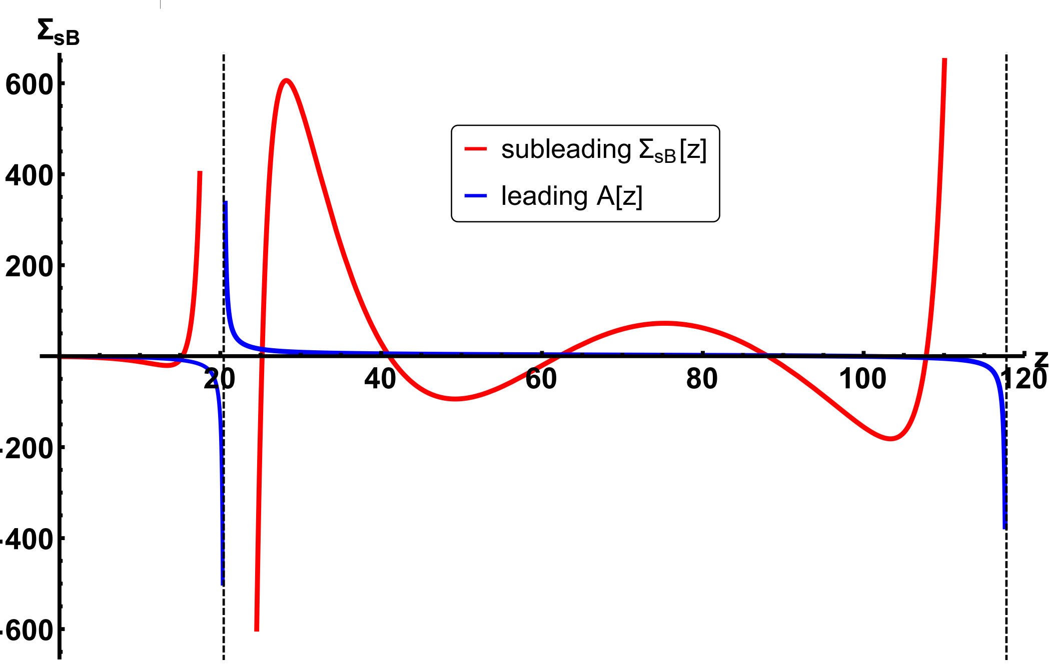
Leading divs



Subleading divs



Infinite number of poles



Infinite number of poles at the same position

Scheme dependence and arbitrariness of subtraction

Divergent counterterms have the form

$$\Sigma_L(z) + \epsilon\Sigma_{NL}(z) + \epsilon^2\Sigma_{NNL}(z) + \cdots$$

$$\Sigma(z) = \sum_n^\infty z^n F_n$$

$$D = 4 \quad N = 4 \quad z = g^2/\epsilon$$
$$D = 6 \quad N = 2 \quad z = g^2 s/\epsilon, z = g^2 t/\epsilon$$
$$D = 8 \quad N = 1 \quad z = g^2 s^2/\epsilon, z = g^2 st/\epsilon, ..$$
$$D = 10 \quad N = 1 \quad z = g^2 s^3/\epsilon, z = g^2 s^2 t/\epsilon, ..$$

All previous eqs. are written in MS scheme

Scheme dependence and arbitrariness of subtraction

Divergent counterterms have the form

$$\Sigma_L(z) + \epsilon\Sigma_{NL}(z) + \epsilon^2\Sigma_{NNL}(z) + \cdots$$

$$\Sigma(z) = \sum_n^\infty z^n F_n$$

$$\begin{array}{rll} D = 4 & N = 4 & z = g^2/\epsilon \\ D = 6 & N = 2 & z = g^2 s/\epsilon, z = g^2 t/\epsilon \\ D = 8 & N = 1 & z = g^2 s^2/\epsilon, z = g^2 st/\epsilon, .. \\ D = 10 & N = 1 & z = g^2 s^3/\epsilon, z = g^2 s^2 t/\epsilon, .. \end{array}$$

All previous eqs. are written in MS scheme

Min Scheme -> Non-Minimal Scheme

Scheme dependence and arbitrariness of subtraction

Divergent counterterms have the form

$$\Sigma_L(z) + \epsilon\Sigma_{NL}(z) + \epsilon^2\Sigma_{NNL}(z) + \cdots$$

$$\Sigma(z) = \sum_n^\infty z^n F_n$$

$$\begin{array}{lll} D=4 & N=4 & z = g^2/\epsilon \\ D=6 & N=2 & z = g^2 s/\epsilon, z = g^2 t/\epsilon \\ D=8 & N=1 & z = g^2 s^2/\epsilon, z = g^2 st/\epsilon, .. \\ D=10 & N=1 & z = g^2 s^3/\epsilon, z = g^2 s^2 t/\epsilon, .. \end{array}$$

All previous eqs. are written in MS scheme

Min Scheme -> Non-Minimal Scheme

$$g^2 \to g'^2 = g^2 + c_1 g^4 + c_2 g^6 + ...$$

Scheme dependence and arbitrariness of subtraction

Divergent counterterms have the form

$$\Sigma_L(z) + \epsilon\Sigma_{NL}(z) + \epsilon^2\Sigma_{NNL}(z) + \cdots$$

$$\Sigma(z) = \sum_n^{\infty} z^n F_n$$

$$\begin{array}{lll} D=4 & N=4 & z = g^2/\epsilon \\ D=6 & N=2 & z = g^2 s/\epsilon, z = g^2 t/\epsilon \\ D=8 & N=1 & z = g^2 s^2/\epsilon, z = g^2 st/\epsilon, .. \\ D=10 & N=1 & z = g^2 s^3/\epsilon, z = g^2 s^2 t/\epsilon, .. \end{array}$$

All previous eqs. are written in MS scheme

Min Scheme -> Non-Minimal Scheme

$$g^2 \to g'^2 = g^2 + c_1 g^4 + c_2 g^6 + ...$$

$$z \to z' = z + c_1 \epsilon z^2 + c_2 \epsilon^2 z^3 + ...$$

Scheme dependence and arbitrariness of subtraction

Divergent counterterms have the form

$$\Sigma_L(z) + \epsilon\Sigma_{NL}(z) + \epsilon^2\Sigma_{NNL}(z) + \cdots$$

$$\Sigma(z) = \sum_n^{\infty} z^n F_n$$

$$\begin{array}{lll} D=4 & N=4 & z=g^2/\epsilon \\ D=6 & N=2 & z=g^2 s/\epsilon, z=g^2 t/\epsilon \\ D=8 & N=1 & z=g^2 s^2/\epsilon, z=g^2 st/\epsilon, .. \\ D=10 & N=1 & z=g^2 s^3/\epsilon, z=g^2 s^2 t/\epsilon, .. \end{array}$$

All previous eqs. are written in MS scheme

Min Scheme -> Non-Minimal Scheme

$$g^2 \to g'^2 = g^2 + c_1 g^4 + c_2 g^6 + ...$$

$$z \to z' = z + c_1 \epsilon z^2 + c_2 \epsilon^2 z^3 + ...$$

Valid both in renormalizable and non-reneormalizable case!

Scheme dependence and arbitrariness of subtraction

Divergent counterterms have the form

$$\Sigma_L(z) + \epsilon\Sigma_{NL}(z) + \epsilon^2\Sigma_{NNL}(z) + \cdots$$

$$\Sigma(z) = \sum_n^\infty z^n F_n$$

$$\begin{array}{lll} D=4 & N=4 & z=g^2/\epsilon \\ D=6 & N=2 & z=g^2 s/\epsilon, z=g^2 t/\epsilon \\ D=8 & N=1 & z=g^2 s^2/\epsilon, z=g^2 st/\epsilon, .. \\ D=10 & N=1 & z=g^2 s^3/\epsilon, z=g^2 s^2 t/\epsilon, .. \end{array}$$

All previous eqs. are written in MS scheme

Min Scheme -> Non-Minimal Scheme

$$g^2 \to g'^2 = g^2 + c_1 g^4 + c_2 g^6 + ...$$

$$z \to z' = z + c_1\epsilon z^2 + c_2\epsilon^2 z^3 + ...$$

Valid both in renormalizable and non-reneormalizable case!

However, interpretation via renormalization of the coupling is lacking in non-renormalizable case

Conclusions

Conclusions

- **The UV divergences for the on-shell scattering amplitudes DO NOT CANCEL in any given order of PT**

Conclusions

- **The UV divergences for the on-shell scattering amplitudes DO NOT CANCEL in any given order of PT**

- **The recurrence relations allow one to calculate the leading UV divergences in ALL orders of PT algebraically starting from 1 loop**

Conclusions

- **The UV divergences for the on-shell scattering amplitudes DO NOT CANCEL in any given order of PT**

- **The recurrence relations allow one to calculate the leading UV divergences in ALL orders of PT algebraically starting from 1 loop**

- **The recurrence relations allow one to calculate the sub leading UV divergences in ALL orders of PT algebraically starting from 1 and 2 loops**

Conclusions

- **The UV divergences for the on-shell scattering amplitudes DO NOT CANCEL in any given order of PT**

- **The recurrence relations allow one to calculate the leading UV divergences in ALL orders of PT algebraically starting from 1 loop**

- **The recurrence relations allow one to calculate the sub leading UV divergences in ALL orders of PT algebraically starting from 1 and 2 loops**

- **This procedure apparently continues the same way for all divergences just like in renormalizable theories**

Conclusions cont'd

Conclusions cont'd

- **The sum of the leading UV divergences to ALL orders obeys the nonlinear integro-differential equation**

Conclusions cont’d

- **The sum of the leading UV divergences to ALL orders obeys the nonlinear integro-differential equation**

- **The numerical solution indicates that solution to the full equation seems to behave like the ladder approximation**

Conclusions cont'd

- **The sum of the leading UV divergences to ALL orders obeys the nonlinear integro-differential equation**

- **The numerical solution indicates that solution to the full equation seems to behave like the ladder approximation**

- **There is no simple limit when** $\epsilon \to +0$

Conclusions cont'd

- **The sum of the leading UV divergences to ALL orders obeys the nonlinear integro-differential equation**
- **The numerical solution indicates that solution to the full equation seems to behave like the ladder approximation**
- **There is no simple limit when** $\epsilon \to +0$
- **This means that one cannot simply remove the UV divergence and non-renormalizability of a theory is not improved when summing the infinite series**

Conclusions cont'd

Conclusions cont'd

- **The structure of UV divergences in non-renormalizable theories essentially copies that of renormalizable ones**

Conclusions cont'd

- **The structure of UV divergences in non-renormalizable theories essentially copies that of renormalizable ones**
- **The arbitrariness of the subtraction procedure is still governed by the redefinition of the «effective» couplings, but in no-renormalizable case they are multiple and depend on kinematics**

Conclusions cont'd

- The structure of UV divergences in non-renormalizable theories essentially copies that of renormalizable ones
- The arbitrariness of the subtraction procedure is still governed by the redefinition of the «effective» couplings, but in no-renormalizable case they are multiple and depend on kinematics
- Establishing connection between these coupling would made the theory tractable in the usual sense