Critical behavior of the Gross-Neveu-Yukawa model at three loops

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based on arXiv:1703.08801

Motivation for GNY model

- $2D$ Dirac materials like graphene [Herbut'06] or cold atom systems: quasi-relativistic Dirac fermions coupled to a bosonic order parameter
- GNY provides UV completion of the Gross-Neveu model in $2 < D < 4$ [Zinn-Justin '91]
  • $1/N_f$ expansions of the two theories at $D = 2 + \epsilon$ and $D = 4 - \epsilon$ coincide
- construction of interacting conformal field theories beyond $D = 2$ [Fei et al '16]
- $N_f = 1/4$ emergent super-symmetry [Zerf et al '16]

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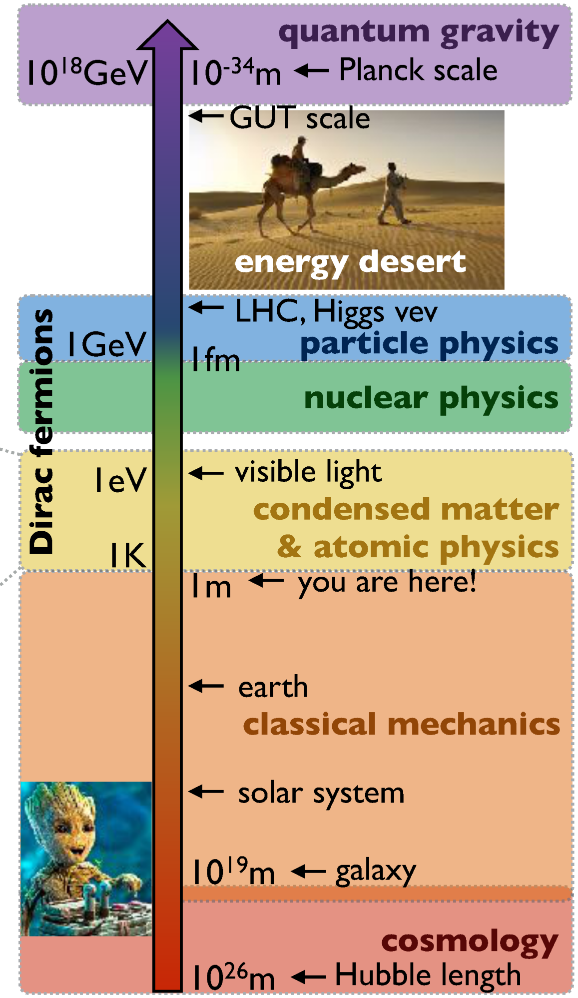
Physics of **scales** - the **renormalization group**

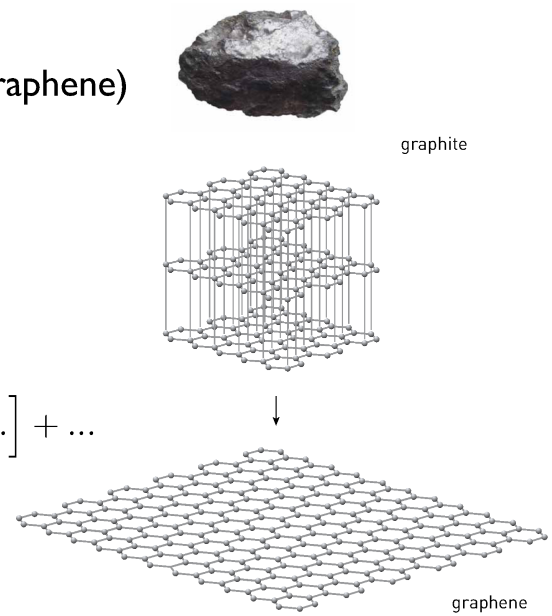
(Functional) Renormalization Group

- Asymptotic safety and quantum gravity
- UV completions of the Standard Model
- Higgs mass and Scale of new physics
- (Strongly-)correlated lattice electrons
- Competing interactions & order parameters
- Phase transitions in Dirac materials
- ultracold atoms & BCS-BEC crossover
- Critical behavior of field theories

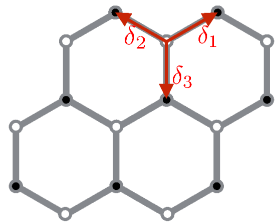


Electrons on the **honeycomb** lattice

- focus on **2D Dirac materials** with electron quasiparticles (graphene)

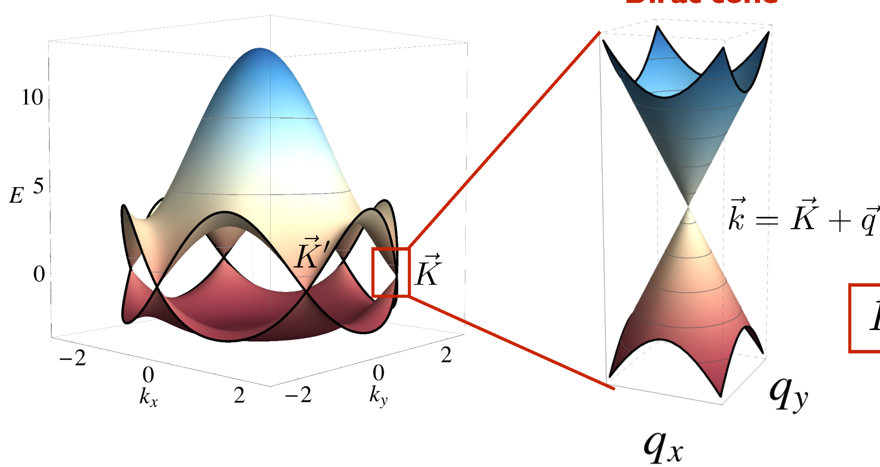


- lattice in real space:



- tight-binding Hamiltonian: $H_0 = -t \sum_{\vec{R},i} \left[u^\dagger(\vec{R}) v(\vec{R} + \vec{\delta}_i) + \text{h.c.} \right] + \ldots$

- energy dispersion: $t = 2.8\ \text{eV}, t' = -0.2t$



$$E_\pm(\vec{q}) \approx 3t' \pm v_F |\vec{q}|$$

- no gap + vanishing density of states → **semimetallic** behavior

Relativistic condensed matter

- honeycomb lattice: 2 different sub-lattices $\rightarrow$ pseudo-spin σ
- quasi particles: 2-component wave functions $\rightarrow$ spinors
- linear spectrum $E = \pm v_F |\vec{k}|$

Charge carriers mimic **relativistic** particles described by Dirac equation

- Action at low energies corresponding to H_0

$$S = \int_0^{1/T} \mathrm{d}\tau \mathrm{d}\vec{x} \sum_{\sigma=\pm 1} \bar{\Psi}_\sigma(\vec{x},\tau)\gamma_\mu \partial_\mu \Psi\sigma(\vec{x},\tau)$$

  - 8-component spinors
  - Clifford algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu,\nu}$
  - γ matrices: $\gamma_0 = I_2 \bigotimes \sigma_z$, $\gamma_1 = \sigma_z \bigotimes \sigma_y$, $\gamma_2 = I_2 \bigotimes \sigma_x$,
    $\gamma_3 = \sigma_x \bigotimes \sigma_y$, $\gamma_5 = \sigma_y \bigotimes \sigma_y$
  - arbitrary number of fermion flavours N

Critical behavior of Ising model

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- 2nd order **phase transition**
- scalar order parameter ϕ
- critical exponents: scaling dimensions of local operators: ϕ^2, ϕ^4

3D Ising universality class:

$$\mathcal{L} \;=\; \frac{1}{2}\phi(m^2 - \partial_\mu^2)\phi + \lambda\phi^4$$

Correlation length: $\xi \sim A|t|^{-\nu}(1 + C|t|^{\omega} + \cdots)\,, \quad t = \frac{T-T_c}{T_c}$

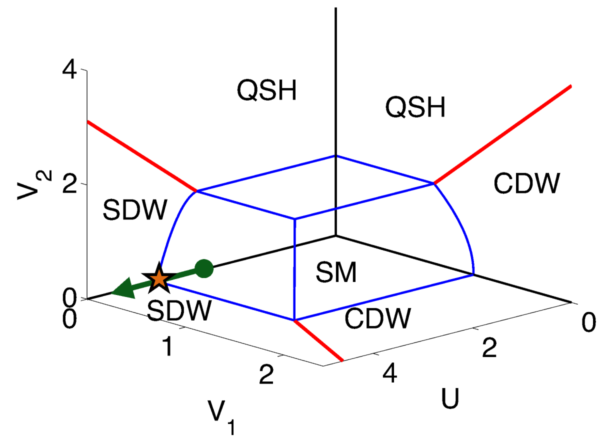
year	Method	ν	η	ω
1998	ϵ-exp	0.63050(250)	0.03650(500)	0.814(18)
1998	3D exp	0.63040(130)	0.03350(250)	0.799(11)
2002	HT	0.63012(16)	0.03639(15)	0.825(50)
2003	MC	0.63020(12)	0.03680(20)	0.821(5)
2010	MC	0.63002(10)	0.03627(10)	0.832(6)
2014	cBS	0.62999(5)	0.03631(3)	0.8303(18)

5-loop calculation in 4-ε dimensions (perturbative RG)

Massless Dirac fermions

- Dirac systems in **(2+1)D** give rise to new universality class

- *e.g.* **anti-ferromagnetic transition**
  - Heisenberg order parameter $\sim O(3)$
  - **massless/gapless** fermion fluctuations

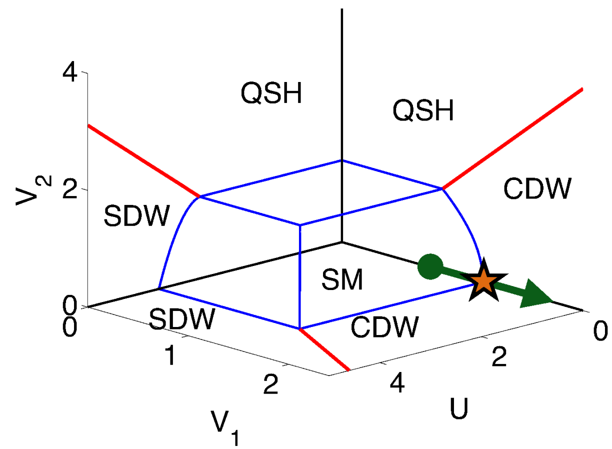


- correlation length critical exponent

Model	N	Class	Method	ν
Honeycomb	8	Chiral Heisenberg	Monte Carlo (present)	1.02(1)
Gross-Neveu	8	Chiral Heisenberg	$4-\epsilon$, first order [31,50]	0.851
Gross-Neveu	8	Chiral Heisenberg	$4-\epsilon$, second order [31,50]	~~1.01~~ **1.08**
Gross-Neveu	8	Chiral Heisenberg	FRG [32]	1.31

[Otsuko, Yunoki, Sorella '16]

Massless Dirac fermions (2)



- *e.g.* **charge density wave** (CDW) transition with Ising OP

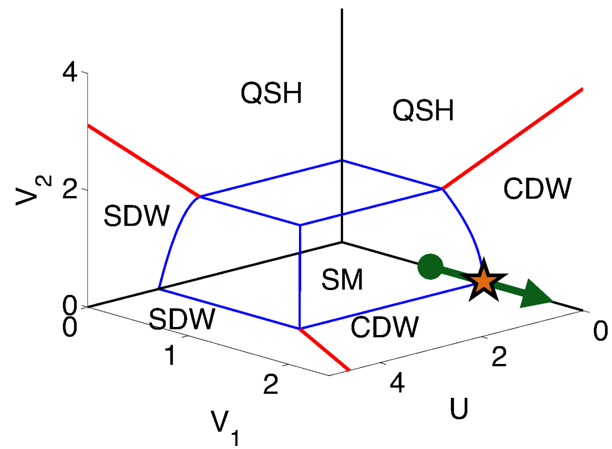
$$\mathcal{L} \quad = \quad \bar{\Psi}(\partial\!\!\!/ + g\phi)\Psi + \frac{1}{2}\phi(m^2 - \partial_\mu^2)\phi + \lambda\phi^4$$

- calculation in $D = 4 - \epsilon$ for arbitrary fermion number N to 3 loops

$$\Psi_0 = \sqrt{Z_\Psi}\Psi\,, \quad \phi_0 = \sqrt{Z_\phi}\phi\,, \quad m_0 = m Z_{\phi^2}/Z_\phi$$
$$\lambda_0 = \mu^\epsilon \lambda Z_{\phi^4}/Z_\phi^2\,, \quad y_0 = \mu^\epsilon y Z_{\phi\bar{\Psi}\Psi}/(Z_\Psi^2 Z_\phi)\,, \quad y = g^2\,.$$

Massless Dirac fermions (2)

- *e.g.* **charge density wave** (CDW) transition with Ising OP



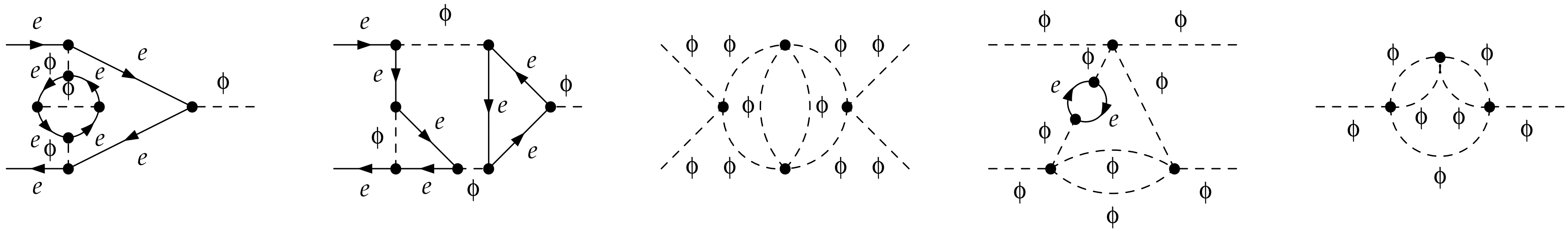
$$\mathcal{L} \quad = \quad \bar{\Psi}(\partial\!\!\!/ + g\phi)\Psi + \frac{1}{2}\phi(m^2 - \partial_\mu^2)\phi + \lambda\phi^4$$

- massive fermions: $m_\Psi \neq 0 \Rightarrow \lambda_3\phi^3$
- $\phi \in SU(N)$, $n > 3$ additional quartic interactions at 3 loops own code to handle the group invariants [N. Zerf]
- UV completion for GN model

$$\mathcal{L} \quad = \quad \bar{\Psi}\partial\!\!\!/\Psi + \frac{g_{GN}}{2}(\bar{\Psi}\Psi)^2$$

Calculation

- $\mathcal{O}(1500)$ Feynman diagrams



- $\overline{\text{MS}}$ scheme
- 2 independent setups
  - IRR: 1 non zero external momentum & all masses set to zero $\Rightarrow$ MINCER [Larin, Tkachov, Vermaseren'91]
  - IR regulator for propagators $\Rightarrow$ MATAD [Steinhauser'00]

Results

$$\beta_x = \frac{dx}{d\ln\mu}\,, \qquad x = y\,,\lambda$$

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$$\beta_x = \frac{dx}{d\ln\mu}\,, \qquad x = y\,,\lambda$$

$$\begin{aligned}
\beta_y = & -\epsilon y + (3+2N)y^2 + 24y\lambda(\lambda - y) - (\frac{9}{8} + 6N)y^3 \\
& + \frac{y}{64}\Big(1152(7+5N)y^2\lambda + 192(91-30N)y\lambda^2 \\
& + \big(912\zeta_3 - 697 + 2N(67 + 112N + 432\zeta_3)\big)y^3 - 13824\lambda^3\Big)\,,
\end{aligned}$$

$$\begin{aligned}
\beta_\lambda = & -\epsilon\lambda + 36\lambda^2 + 4Ny\lambda - Ny^2 + 4Ny^3 + 7Ny^2\lambda \\
& - 72Ny\lambda^2 - 816\lambda^3 + \frac{1}{32}\Big(6912(145 + 96\zeta_3)\lambda^4 \\
& + 49536Ny\lambda^3 - 48N(72N - 361 - 648\zeta_3)y^2\lambda^2 \\
& + 2N(1736N - 4395 - 1872\zeta_3)y^3\lambda \\
& + N(5 - 628N - 384\zeta_3)y^4\Big)\,.
\end{aligned}$$

Results

$$\beta_x = \frac{dx}{d\ln\mu}\,, \qquad x = y\,,\lambda$$

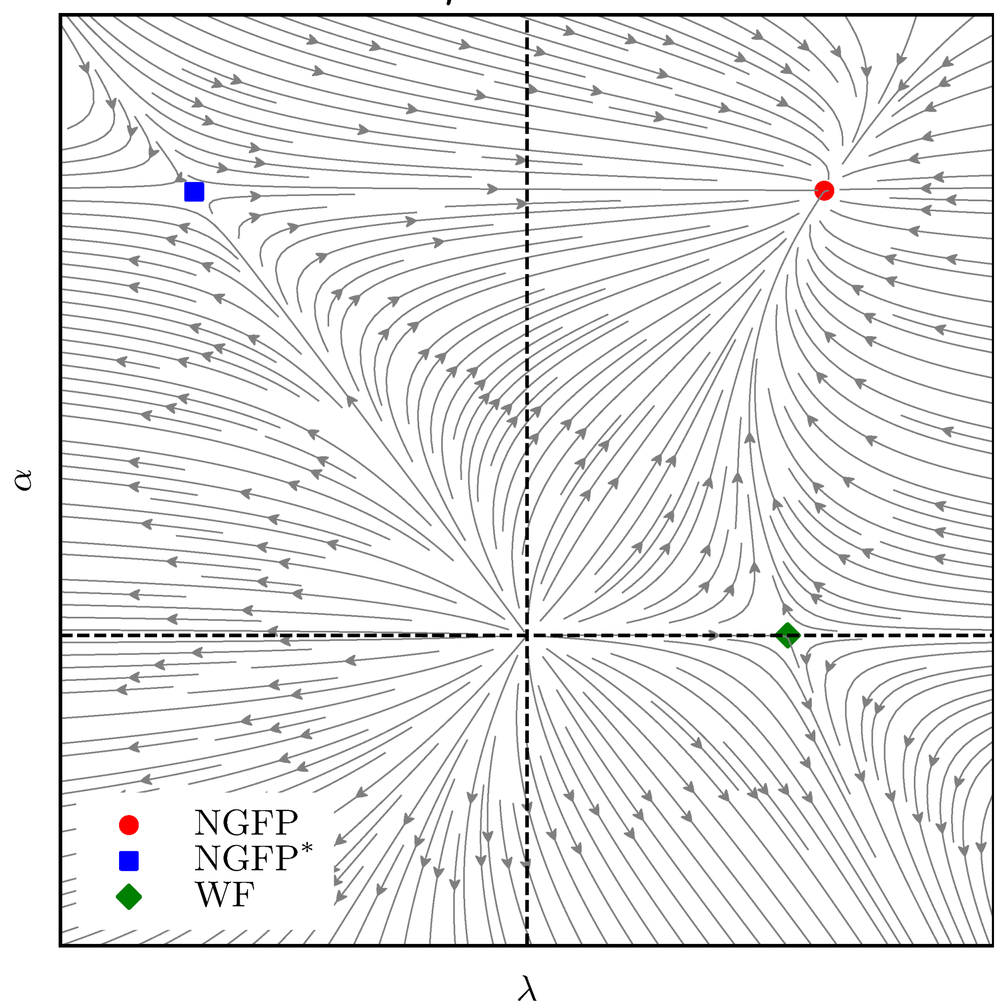
- Fixed points: $\beta_y(y_*, \lambda_*) = 0\,, \quad \beta_\lambda(y_*, \lambda_*) = 0\,,$
  - unstable Gaußian fixed point $(y_*, \lambda_*)_0 = (0, 0)$
  - unstable Wilson-Fischer fixed point $(y_*, \lambda_*)_{\mathrm{WF}} = (0, \epsilon/36)$
  - non-Gaußian fixed points (NGFP)

$$(y_*, \lambda_*)_\pm = \left(\frac{1}{3+2N}\,\epsilon, \frac{3-2N \pm s}{72(3+2N)}\,\epsilon \right)\,,$$

$$s = \sqrt{9 + 4N(33+N)}\,.$$

Results

$$\beta_x = \frac{dx}{d\ln\mu}\,, \quad x = y\,,\lambda$$



Critical exponents in $D = 4 - \epsilon$

$$\gamma_x = \frac{d \ln Z_x}{d \ln \mu} \quad \text{for} \quad x \in \{\Psi, \phi, \phi^2\}$$

- NGFP:

$$\begin{aligned} \eta_\psi &= \gamma_\psi(y_*, \lambda_*), \\ \eta_\phi &= \gamma_\phi(y_*, \lambda_*), \\ \eta_{\phi^2} &= \gamma_{\phi^2}(y_*, \lambda_*), \\ \nu^{-1} &= 2 - \eta_\phi + \eta_{\phi^2} \end{aligned}$$

Critical exponents in $D = 4 - \epsilon$

$$\gamma_x = \frac{d \ln Z_x}{d \ln \mu} \quad \text{for} \quad x \in \{\Psi, \phi, \phi^2\}$$

$$\begin{aligned}
\frac{1}{\nu} &= 2 - \frac{(3+10N+s)\epsilon}{6(3+2N)} \\
&- \frac{513 - 7587N - 666N^2 - 5264N^3 - 96N^4 + s(171 + 510N + 436N^2 + 48N^3)}{108(3+2N)^3 s}\epsilon^2 \\
&+ \frac{\epsilon^3}{3888(3+2N)^5 s^3}\Big(- 227691(3+s) + 4N(81(2170s - 128871) + N(27(-2238507 + 554816s) \\
&+ 2N(585(2414s - 16143) + N(4N(5233698 + N(1383001 + 16N(3832 + 54N - 27s) - 24986s) - 371936s \\
&- 3(8117973 + 761116s))))) + 288(3+2N)s^2(2N(81 + N(1917 + 4N(450 + N(153 + 4N)))) \\
&- N(3+4N)(99 + 4N(21+N))s + 81(3+s))\zeta_3\Big) .
\end{aligned}$$

$$\begin{aligned}
\eta_\psi &= \frac{\epsilon}{2(3+2N)} + \frac{180 + 33s + N(3 - 328N + 2s)}{216(3+2N)^3}\epsilon^2 \\
&+ \Big(\frac{102519 + 237519N + 342N^2 - 122020N^3 - 11040N^4}{7776(3+2N)^5} \\
&- \frac{68607 + 2099304N + 1629828N^2 + 1505352N^3 + 89536N^4 + 1248N^5}{7776(3+2N)^5 s} - \frac{6(1+N)\zeta_3}{(3+2N)^4}\Big)\epsilon^3 ,
\end{aligned}$$

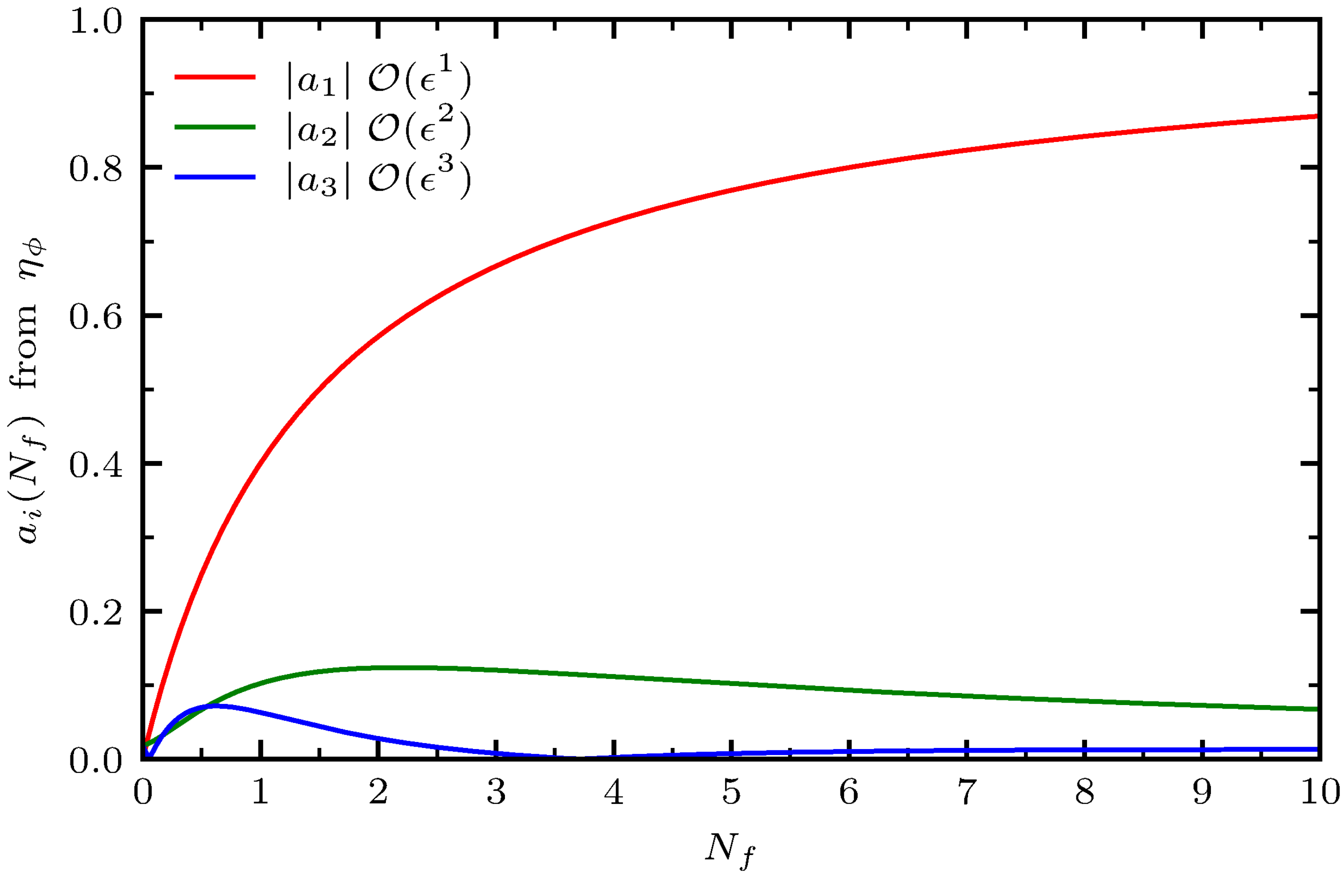
Critical exponents in $D = 4 - \epsilon$

$$\gamma_x = \frac{d \ln Z_x}{d \ln \mu} \quad \text{for} \quad x \in \{\Psi, \phi, \phi^2\}$$

$N = 2:$	ν^{-1}	$\approx$	$2 - 0.952\epsilon + 0.00723\epsilon^2 - 0.0949\epsilon^3$
	η_ψ	$\approx$	$0.0714\epsilon - 0.00671\epsilon^2 - 0.0243\epsilon^3$
	η_ϕ	$\approx$	$0.571\epsilon + 0.124\epsilon^2 - 0.0278\epsilon^3$
$N = 1:$	ν^{-1}	$\approx$	$2 - 0.835\epsilon - 0.00571\epsilon^2 - 0.0603\epsilon^3$
	η_ψ	$\approx$	$0.1\epsilon + 0.0102\epsilon^2 - 0.033\epsilon^3$
	η_ϕ	$\approx$	$0.4\epsilon + 0.102\epsilon^2 - 0.0632\epsilon^3$
$N = 1/4:$	ν^{-1}	$\approx$	$2 - 0.571\epsilon - 0.0204\epsilon^2 + 0.024\epsilon^3$
	η_ψ	$\approx$	$0.143\epsilon + 0.0408\epsilon^2 - 0.048\epsilon^3$
	η_ϕ	$\approx$	$0.143\epsilon + 0.0408\epsilon^2 - 0.048\epsilon^3$
$N = 0:$	ν^{-1}	$\approx$	$2 - 0.333\epsilon - 0.117\epsilon^2 + 0.125\epsilon^3$
	η_ψ	$\approx$	$0.167\epsilon + 0.0478\epsilon^2 - 0.0469\epsilon^3$
	η_ϕ	$\approx$	$0.0185\epsilon^2 + 0.0187\epsilon^3$

Critical exponents in $D = 4 - \epsilon$

$$\gamma_x = \frac{d \ln Z_x}{d \ln \mu} \quad \text{for} \quad x \in \{\Psi, \phi, \phi^2\}$$



Critical exponents in $D = 4 - \epsilon$

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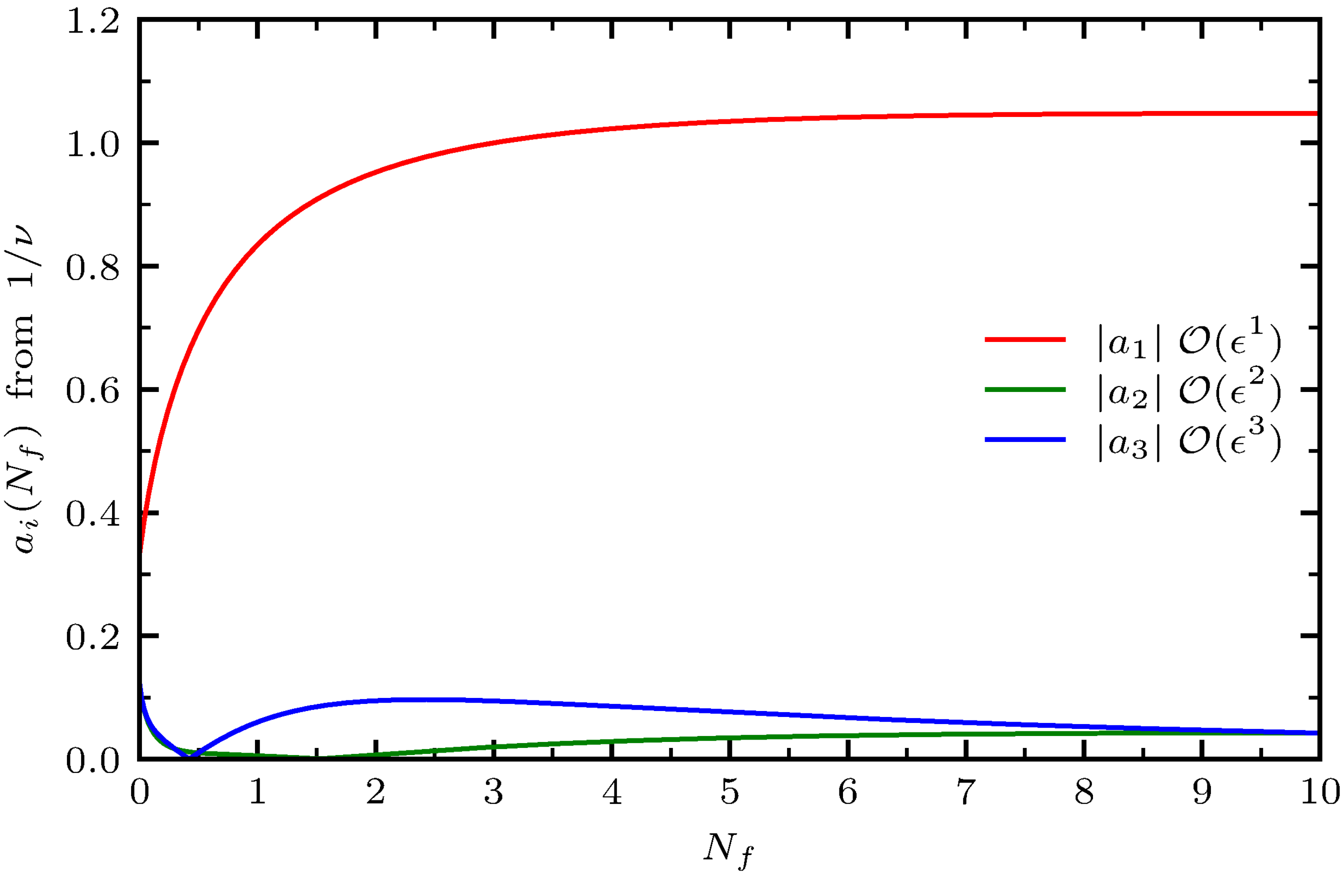

Critical exponents in $D = 3$

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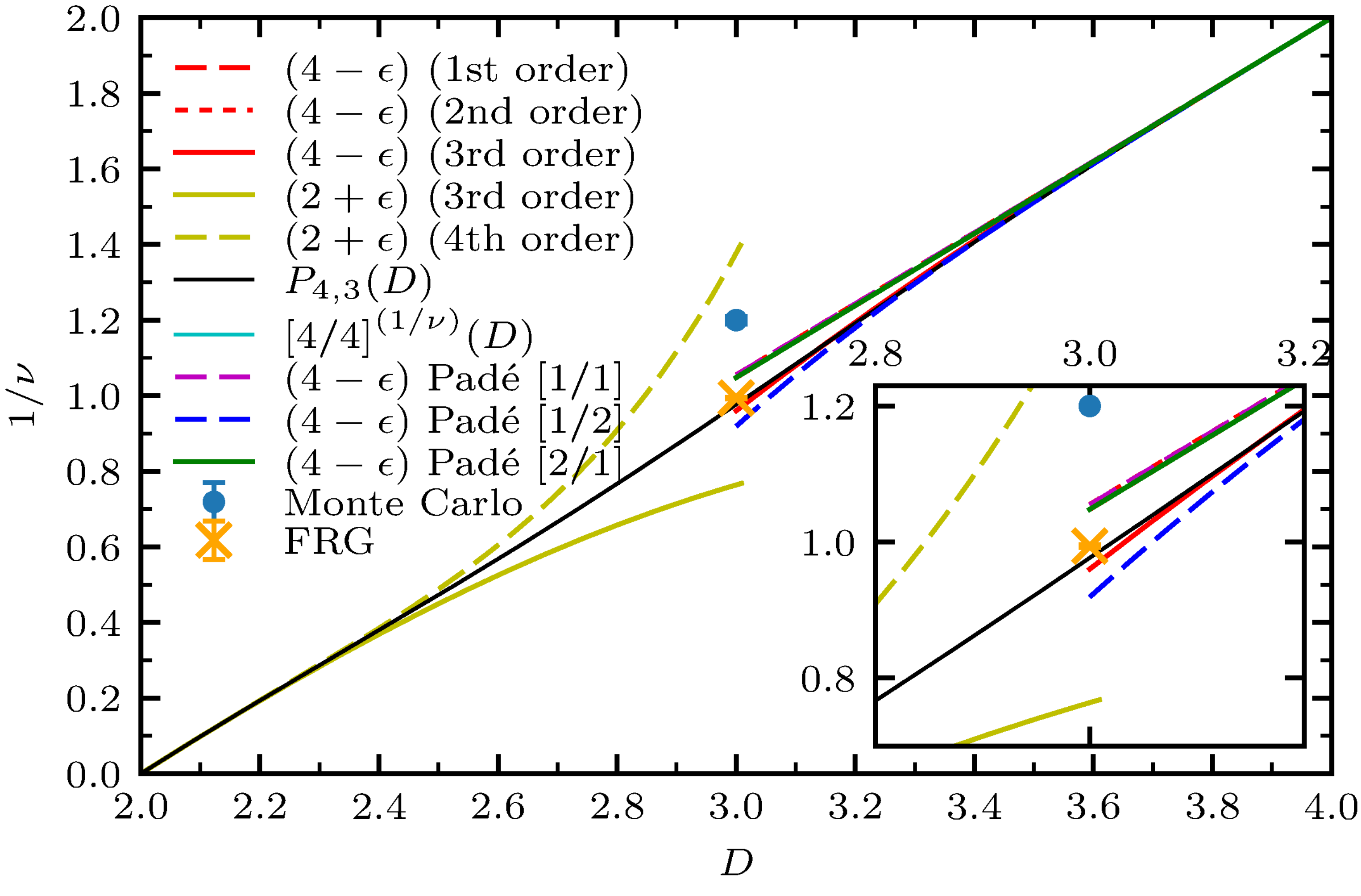
- Padé approximants for $D = 4 - \epsilon$ results

$N = 2$	$1/\nu$	η_ϕ	η_ψ
this work (Padé $[2/1]$)	1.048	0.672	0.0740
$(2+\epsilon)$, (ϵ^4, Padé)	0.931	0.745	0.082
functional RG	0.994(2)	0.7765	0.0276
Monte Carlo	1.20(1)	0.62(1)	0.38(1)
$N = 1$	$1/\nu$	η_ϕ	η_ψ
this work (Padé $[2/1]$)	1.166	0.463	0.102
functional RG	1.075(4)	0.5506	0.0645
Monte Carlo	1.30	0.45(3)	
$N = 1/4$	$1/\nu$	η_ϕ	η_ψ
this work (Padé $[2/1]$)	1.419	0.162	0.162
functional RG	1.408	0.180	0.180
conformal bootstrap		0.164	0.164

Critical exponents for $N = 2$

- Padé approximants or polynomial interpolation for $D = 4 - \epsilon$ and $2 + \epsilon$ [Gracey et al '16] results



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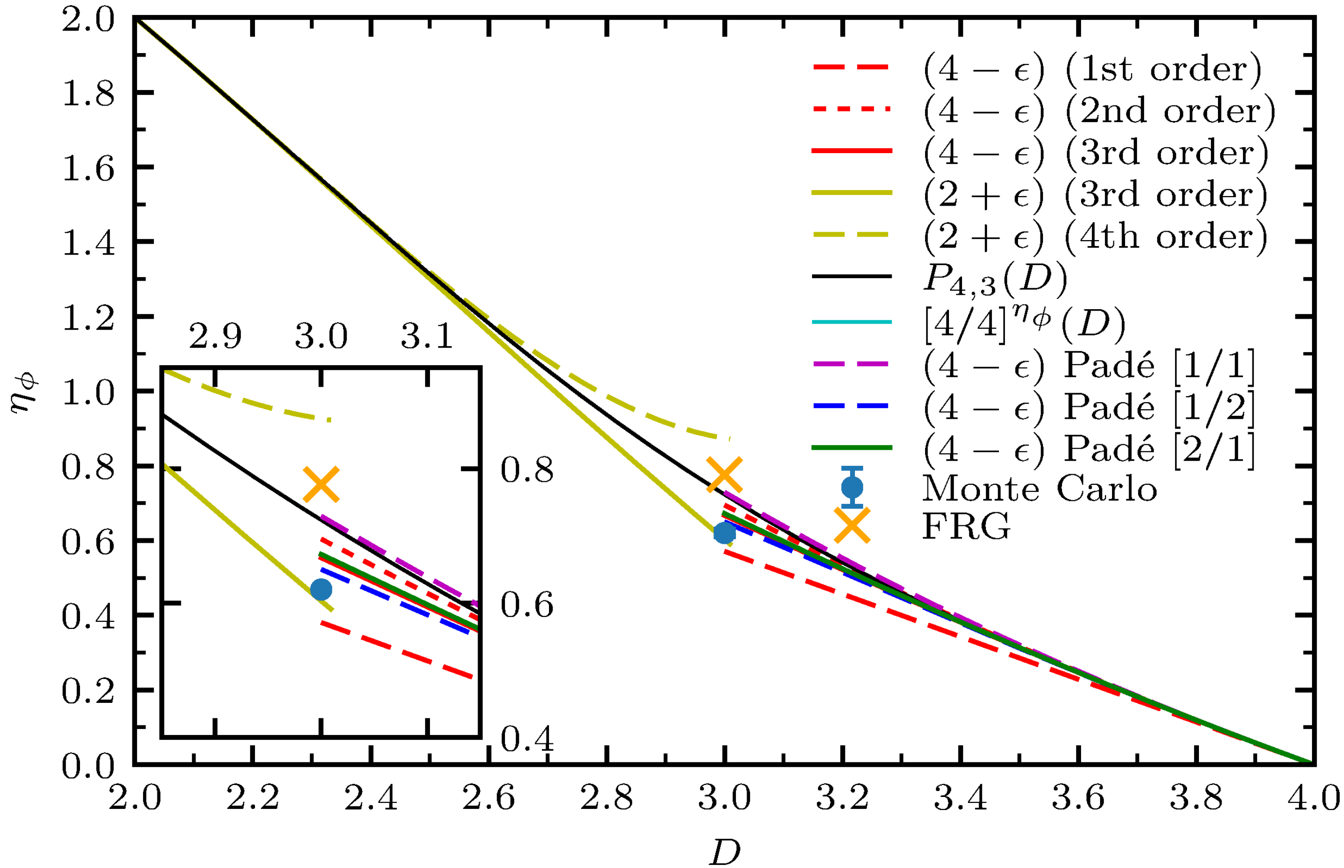

Outlook

- Resummation of ϵ expansion:
  Borel-sum, Padé-Borel-sum, conformal mapping
  Is the large order behavior of GNY known?
- Higher order contribution for the ϵ expansion: 4-loop calculation
- Other order parameters: chiral Heisenberg model, XY model