7 loops φ^4

78.15% come from primitives DB.

MS dimreg φ^4: $u=1$

$$\beta(g) = 3g^2 - \frac{17}{3}g^3 + \ldots + \left(\frac{195\,654\,263}{23040} + \frac{15676169}{720}\zeta(3) + \ldots \right.$$

26 terms

$$\left. + 24 P_{7,11} \right) g^8$$

MZV

E. Panzer

6th roots of unity

$$\approx 474\,651\, g^8.$$

$$\gamma(g) = \frac{1}{12}g^2 \ldots - 124.159\, g^7$$

$$\gamma_u(g) = -g + \ldots - 137598\, g^7$$

$\Sigma(p)$ to 6 loops.

78.15%
come from
primitives
DB.

$\zeta(3) + \ldots$

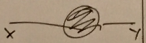
g^8

roots of unity

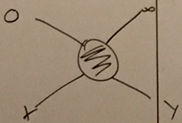
The method

Position space

- Bare theory
  $D = 4 - \varepsilon$ dimensions
- calculate 1 scale processes

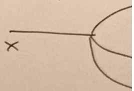


2 point fct.

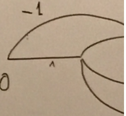


- graph
- Gene
  hy

4 loop.



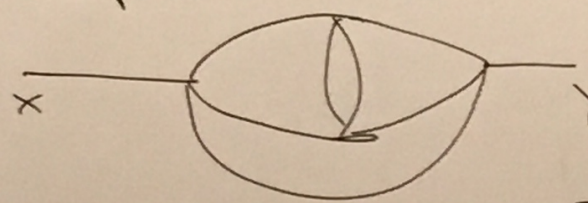
$f(z) =$



$c = f($

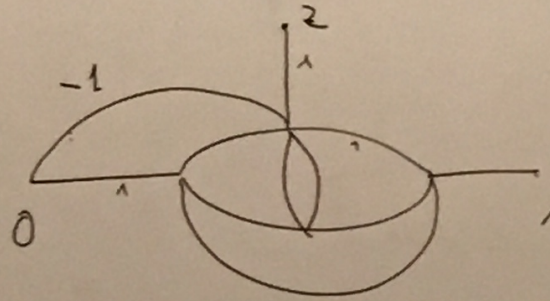
- Graphical functions
- Generalized single-valued hyperlogarithms.

4 loop.

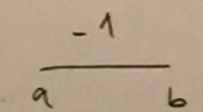


Ordre ε^2

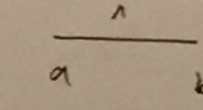
$= \left(C(\varepsilon) \cdot \|x-y\|^{\#}\right)$.



$c = f(0)$



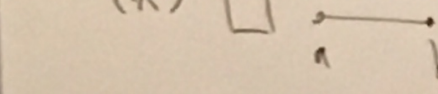
$= (a-b)^{2(1-\varepsilon/2)}$



$= (a-b)^{-2(1-\varepsilon/2)}$

graphical function

- $\mathcal{L}(z)$ can be considered as a real analytic Diagh-related function $\mathbb{P}^1(\mathbb{C}) \setminus \{0,1,\infty\}$ (M. Golz, E. Panzo, OS)

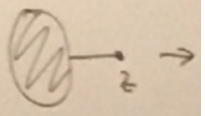
(*) 

$\sim \delta(a-b)$

- Thm: (OS)
  On the space of graphical functions the differential equations (*) can **always** be solved **uniquely**.

Reduction steps

(a) permute the vertices $\{0,1$

(b) 

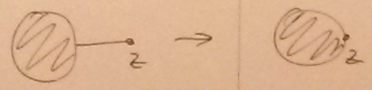
→

(c) edges bet
vertices an
and can

$\overline{0\ z}$ =

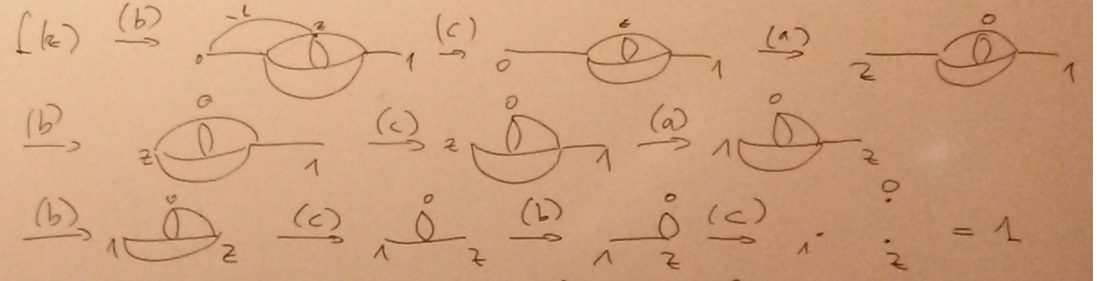
Reduction steps

(a) permute the external vertices $\{0,1,z\}$

(b) 

(c) edges between external vertices are trivial and can be removed

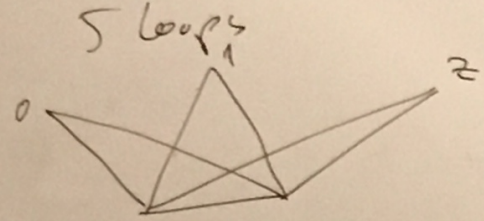
$$\overset{0}{\bullet}\!\!-\!\!\bullet\overset{z}{} = \frac{1}{(z\bar{z})^{1-\varepsilon/2}}$$

[k] 

Compute Maple hyperlog-procedures 9 sec (1 core)

4 loops

betagamma(4,4) 28 core sec



O(ε) trick (completion)

(d) Tricks that work only to some order in ε

5 loops 8 core minutes Office PC

6 loops

(e) Parametric integration (F. Brown, E. Panzer)

6 loops 5 core hours 6 GB Ram.

7 loops

work horse

7 loops ≈ 3 core weeks ≈ 48 GB Ram