Workshop: "Multi-loop calculations (methods and applications)"

Paris, France

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Conformal Feynman graphs from integrable chiral CFT

Vladimir Kazakov (ENS, Paris)

Collaborations with

J. Caetano and Ö. Gürdogan (arXiv:1512.06704 and arXiv:1612.05895)

N. Gromov, G. Korchemsky, S. Negro, G. Sizov (arXiv:1706…..)

D. Chicherin, F. Loebbert, D. Mueller, D. Zhang (arXiv:1704.01967)

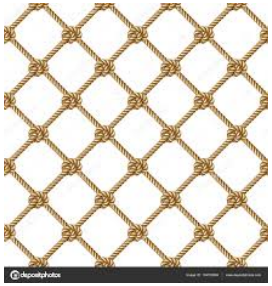
J. Caetano, B. Eden, O. Gurdogan, V. Smirnov (arXiv:1707…)

Outline

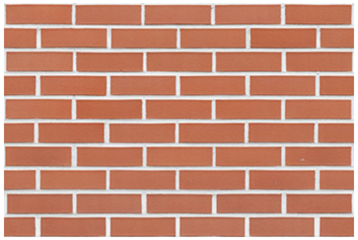
- Integrability of planar N=4 SYM (including integrable deformations) allows to compute interesting physical quantities up to a very high loop order or even exactly:
- Anomalous dimensions computable via Quantum Spectral Curve (QCS), structure constants (hexagon decomposition), correlators, qq-potential, amplitudes, etc.
- Can this integrability compute individual 4D Feynman graphs?

- Double scaling limit of N=4 SYM: strong γ-deformation & weak coupling leads to non-unitary, “chiral” CFT dominated by integrable (computable!)   graphs

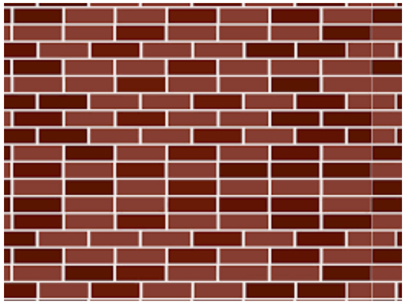
“fishnet” A. Zamolodchikov 1980
(4-scalar interaction)



“brick wall” Gurdogan, V.K. 2015
Caetano, Gurdogan, V.K. 2016
(Yukawa interaction)



mixture of both



- Very limited sets of graphs for each loop order: for many quantities one or none. Computing such quantity we compute directly the Feynman graphs
- Basis of integrability: conformal SU(2,2) spin chain: a window to full AdS/CFT integrability
- 3D twisted ABJM model in double scaling limit: regular triangular graphs
- We compute anomalous dimensions of BMN-type operators dominated by wheel graphs

γ-twisted N=4 SYM and double scaling limit

$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^\mu\phi_i^\dagger D_\mu\phi^i + i\bar\psi_A^{\dot\alpha} D_{\dot\alpha}^{\alpha}\psi_\alpha^A] + \mathcal{L}_{\mathrm{int}}$$

$$\mathcal{L}_{\mathrm{int}} = N_c g^2 \;\mathrm{tr}\left(\frac{1}{4}\{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger, \phi^j\} - e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j\right) +$$

$$N_c\, g\, \mathrm{tr}(-e^{-\frac{i}{2}\gamma_j^-}\bar\psi_j\phi^j\bar\psi_4 + e^{+\frac{i}{2}\gamma_j^-}\bar\psi_4\phi^j\bar\psi_j + i\epsilon_{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\bar\psi^k\phi^i\bar\psi^j + \text{conjugate terms}).$$

$$\gamma_1^\pm = -\frac{\gamma_3 \pm \gamma_2}{2}$$

$$\gamma_2^\pm = -\frac{\gamma_1 \pm \gamma_3}{2}$$

$$\gamma_3^\pm = -\frac{\gamma_2 \pm \gamma_1}{2}$$

- γ-twisted N=4 SYM Lagrangian: product of matrix fields → star-product

$$A\, B \to A \star B \equiv q_{A,B}\, A\, B$$

$$q_{A,B} = e^{-\frac{i}{2}\epsilon^{mjk}\gamma_m\, J_j^A\, J_k^B} = (q_{B,A})^{-1}$$

$J_1^A,\ J_2^A,\ J_3^A$ R – charges

(twists : $\gamma_1, \gamma_2, \gamma_3$)

Leigh, Strassler
Frolov
Beisert, Roiban
Lunin, Maldacena

- Double scaling limit: strong twist, weak coupling

Gurdogan, V.K. 2015

$$g \to 0, \qquad e^{-i\gamma_j/2} \to \infty, \qquad \xi_j = g\, e^{-i\gamma_j/2} - \text{fixed}, \qquad (j = 1,2,3.)$$

$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{2}\partial^\mu\phi_i^\dagger\partial_\mu\phi^i + i\bar\psi_A^{\dot\alpha}\partial_{\dot\alpha}^{\alpha}\psi_\alpha^A] + \mathcal{L}_{\mathrm{int}}$$

$$\mathcal{L}_{\mathrm{int}} = N_c\, \mathrm{tr}[\xi_1^2\, \phi_2^\dagger\phi_3^\dagger\phi_2\phi_3 + \xi_2^2\, \phi_3^\dagger\phi_1^\dagger\phi_3\phi_1 + \xi_3^2\, \phi_1^\dagger\phi_2^\dagger\phi_1\phi_2 +$$

$$+ i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2 + \bar\psi_3\phi_1^\dagger\bar\psi_2) + i\sqrt{\xi_1\xi_3}(\psi^1\phi^2\psi^3 + \bar\psi_1\phi_2^\dagger\bar\psi_3) + i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1 + \bar\psi_2\phi_3^\dagger\bar\psi_1)\,].$$

- Breaks all supersymmetry and R-symmetry: PSU(2,2|4)→SU(2,2)×U(1)^3

Gurdogan, V.K. 2015

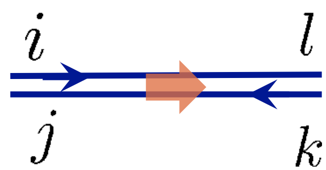
Special case: bi-scalar chiral "CFT"

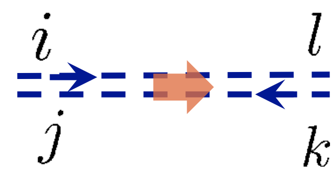
- **Special case:** $\xi := \xi_3$ – fixed, $\xi_1 = \xi_2 = 0$

$$\mathcal{L}[\phi_1, \phi_2] = \frac{N_c}{2} \mathrm{tr}\ \left(\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + 2\xi^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2 \right) .$$

Zero dimensional analogue:
Kostov, Staudacher 1995

- **Propagators**

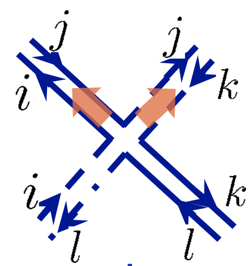




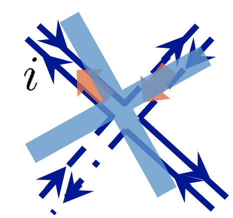
$i, j, k, l = 1 \ldots N_c$

$$\left\langle \phi_1^{*ij}(y)\phi_1^{kl}(x) \right\rangle_0 = \left\langle \phi_2^{*ij}(y)\phi_2^{kl}(x) \right\rangle_0 = \delta^{il}\delta^{jk} \frac{1}{(x-y)^2}$$

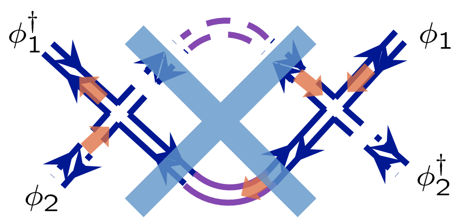
- **Vertex:** $\xi^2 \mathrm{tr}\,(\phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2)$

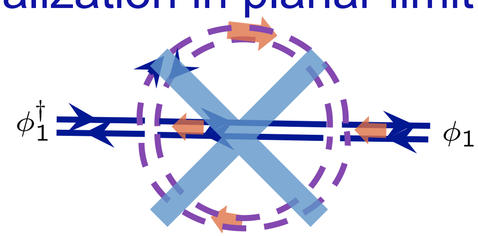


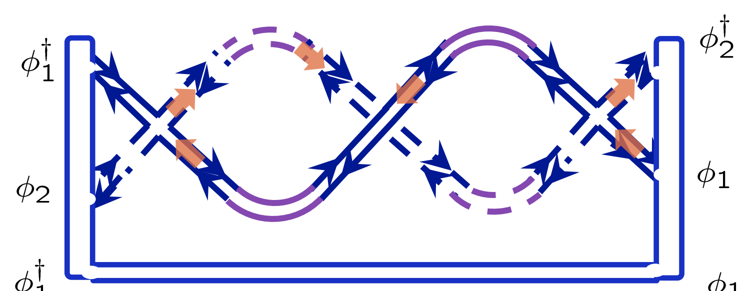
Missing
"anti-chiral" vertex



- Very limited number of planar graphs
- No mass or vertex renormalization in planar limit!







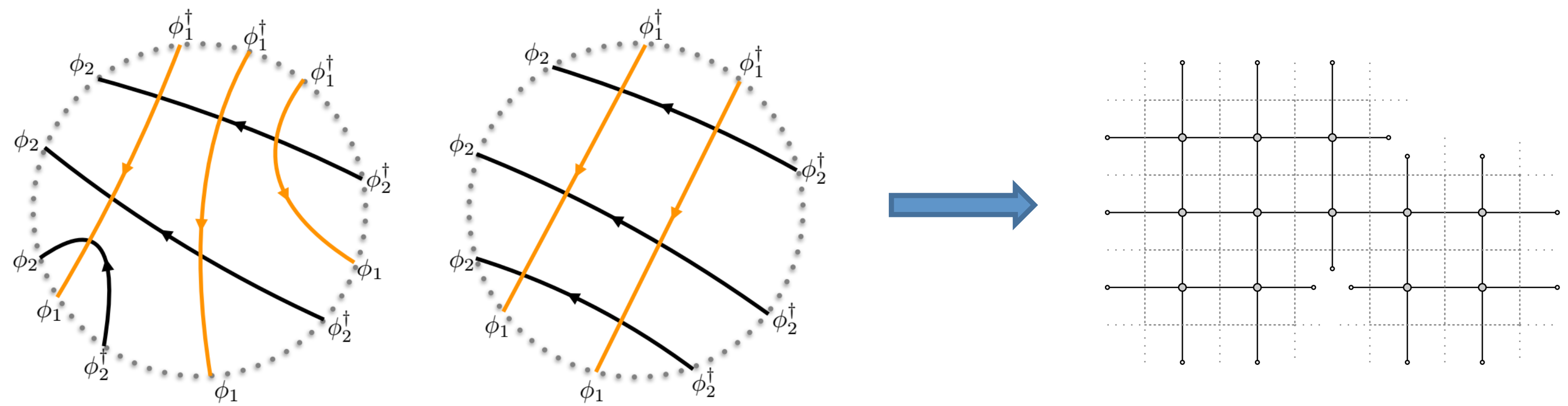
$\mathrm{tr}(\phi_1^\dagger \phi_2)\mathrm{tr}(\phi_2^\dagger \phi_1)$

- ξ does not run in planar limit. But $\mathrm{tr}^2(\phi\, \phi^+)$ do run -- conformal anomaly!

Fishnet amplitudes in bi-scalar model

- Single-trace correlator defined by a single multi-loop "fishnet" graph

$$K(x_1, x_2, \ldots x_{2M}) = \langle \mathrm{Tr}\,[\chi_1(x_1)\,\chi_2(x_2), \ldots \chi_{2M}(x_{2M})] \rangle\,, \qquad \chi_i \in \{\,\phi_1^\dagger, \phi_2^\dagger, \phi_1, \phi_2\}$$

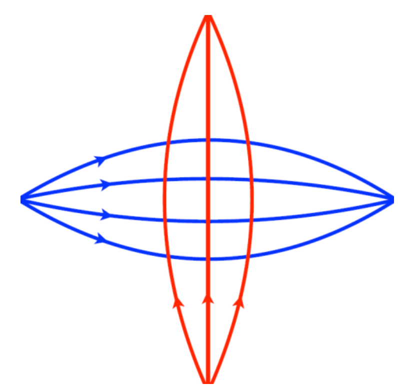


Chicherin, V.K., Loebbert, Mueller, Zhong 2017

- Yangian symmetry produces new PDE's defining these graphs (talk of D. Chicherin)

- Particular case, 4-point fnction, computed explicitly by conformal bootstrap

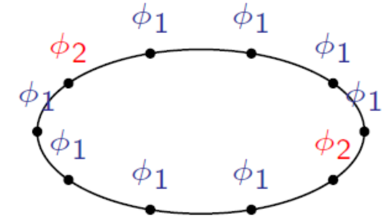
Basso, Dixon 2017



Caetano, Gurdogan, V.K , 2016

Operators, correlators, graphs…

- Operators with M magnons ϕ_2 in the "vacuum" of ϕ_1

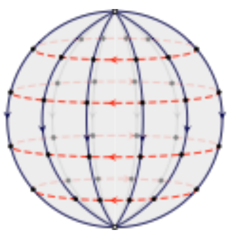


$$\mathcal{O}_{L,M}(x) = \mathrm{tr}\left(\underbrace{\phi_2\phi_1\phi_1\phi_1\phi_2\ldots\phi_1}_{L\ \text{fields}}\right)(x) + \text{permutations}$$

- Correlators $\langle \mathcal{O}(x)\ \mathcal{O}(0)\rangle \sim |x|^{-2L-2\gamma(\xi)}$ anomalous dimension

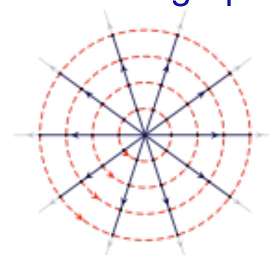
$\mathrm{tr}[\phi_1(x)]^L$

BMN vacuum (non-protected)

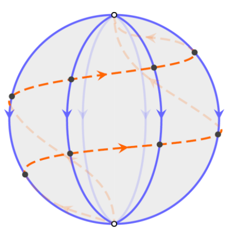


UV- reduction

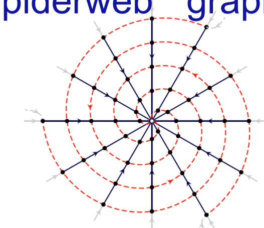
"wheel" graphs



Multi-magnon spiral graphs



"spiderweb" graphs

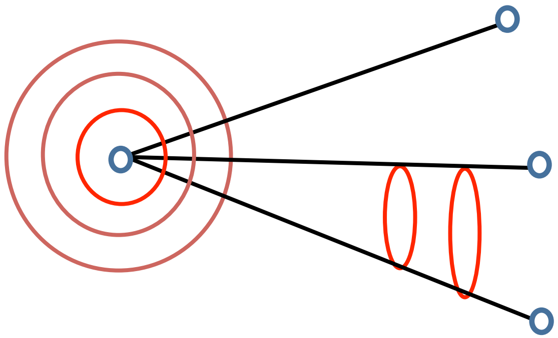


Typical "fishnet" structure in the bulk. Integrable!

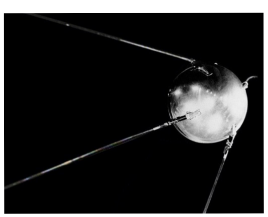
Caetano, Eden, Gurdogan, V.K , V.Smirnov
(to appear)

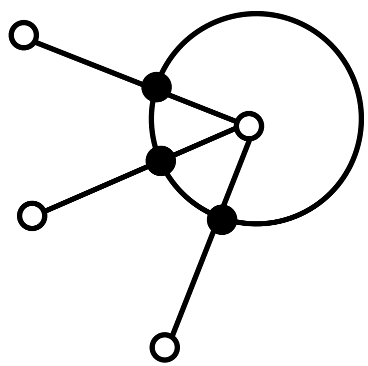
4-point function and “Sputnik” diagram

- Simplest 4-point function given by graphs

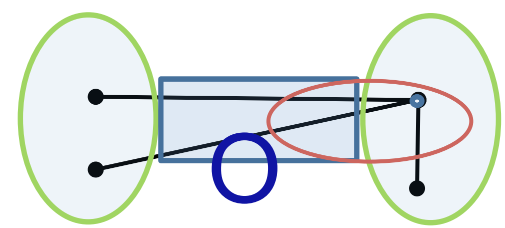


- “Sputnik” - the single 4-point 3-loop conformal graph for given operators
- The graph computed in terms of poly-logs (talk of V.Smirnov)

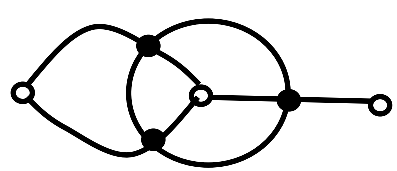




$$= \Sigma_O$$



- Can be used for extracting structure constants from OPE.
- Example:



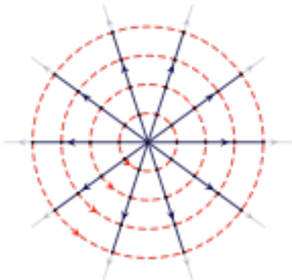
$$\Longrightarrow \quad C_{\mathrm{tr}\phi_1,\mathrm{tr}\phi_1^2,\mathrm{tr}\phi_1^{\dagger 3}} = 6\zeta_3$$

Gonsalves
Basso

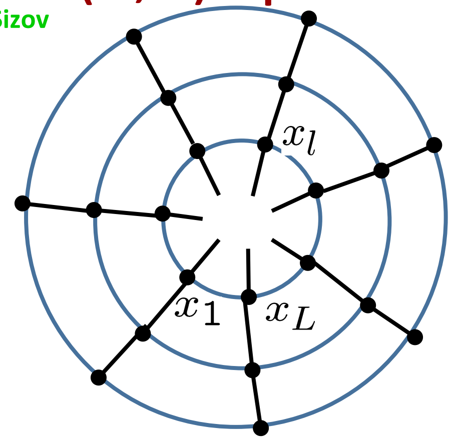
Fishnet graphs and integrable conformal SU(2,2) spin chain

Gromov, V.K , Korchemsky, Negro, Sizov
(to appear)

- "Hamiltonian" generating a wheel (fishnet) graph



$$\hat{H}_L = \xi^{2L} \prod_{l=1}^{L} \frac{1}{(x_{l+1}-x_l)^2} \prod_{l=1}^{L} \Box_{x_l}^{-1}$$



- Conformal symmetry easily checked. E.g. it is covariant w.r.t. inversion:

$$I[\hat{H}_L] = (x_1^2 \ldots x_L^2)\, \hat{H}_L\, (x_1^2 \ldots x_L^2)^{-1}$$

- Integrability: graph-generating Hamiltonian commutes with T-matrix built on conformal group in 4D:

$$[\hat{H}_L, \hat{T}_L(u)] = 0, \qquad \hat{T}_L(u) = \mathrm{tr}[\mathcal{L}_1(u_+,u_-)\,\mathcal{L}_2(u_+,u_-)\ldots\mathcal{L}_L(u_+,u_-)]$$

with Lax operator $\mathcal{L}(u_+,u_-) = \begin{pmatrix} 1 & 0 \\ \mathbf{x} & 1 \end{pmatrix} \begin{pmatrix} u_+\cdot\mathbf{1} & \mathbf{p} \\ \mathbf{0} & u_-\cdot\mathbf{1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\mathbf{x} & 1 \end{pmatrix}$

$$u_+ \equiv u + \frac{\Delta - 4}{2}, \qquad u_- \equiv u - \frac{\Delta}{2}$$

$$\mathbf{x} \equiv -i\bar{\boldsymbol{\sigma}}_\mu x_\mu, \qquad \mathbf{p} \equiv -\frac{i}{2}\boldsymbol{\sigma}_\mu \partial_{x_\mu}$$

D.Chicherin, S.Derkachov, A.Isaev

- "Hamiltonian" emerges at special value of transfer-matrix in principal series auxiliary representation Gromov, V.K , Korchemsky, Negro, Sizov

- Hence integrability of bi-scalar model is demonstrated explicitly in all orders!

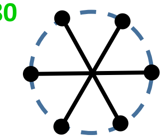
Wheel graphs from dimension of BMN vacuum $\mathrm{tr}[\phi_1(x)]^L$

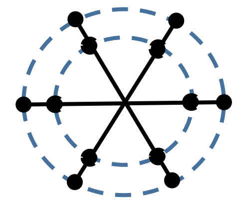
- Anom. dimension at M wrappings = (period of) wheel graph with M frames

$$\gamma_{\mathrm{vac}}(L) = -2\binom{2L-2}{L-1}\zeta_{2L-3}\,\xi^{2L} + \gamma^{(2)}_{\mathrm{vac}}(L)\,\xi^{4L} + \mathcal{O}(\xi^{6L}),$$

Gurdogan, V.K. 2015
(and discussion with Sieg, Wilhelm)

Broadherst 1980





$$\gamma^{(2)}_{\mathrm{vac}}(3) = 2^{-11}\left(\frac{189}{2}\zeta_7 - 72\zeta_3^2\right)$$

(confirmed by direct graph computation!) E.Panzer, 2015

$$\gamma^{(2)}_{\mathrm{vac}}(4) = 2^{-15}\,8\left[\frac{309}{4}\zeta_{11} + 4\zeta_{3,8} + 5\zeta_{5,6} - \zeta_{6,5} + 10\zeta_{8,3} - 2\zeta_{3,3,5} + 10(\zeta_{3,5,3} + \zeta_{5,3,3}) - 50\,\zeta_5^2\right]$$

Rieman multi-zeta numbers

$$\begin{aligned}
\gamma^{(2)}(L) = \frac{2^3}{\Gamma^2(L)}\Bigg\{ &-\sum_{j_1=0}^{L-1}\frac{(2L-2)!}{L-j_1}\binom{L+j_1-1}{j_1}\zeta_{2L-3}\zeta_{\substack{L+j_1-2,\\ L-j_1}} + \frac{\Gamma^2(L)}{8}\binom{4L-2}{2L-1}\zeta_{4L-5}\\
&+\sum_{\substack{j_1,j_2>0\\ j_1+j_2<2L-3}}\frac{\Gamma(2L-j_1-j_2-1)\Gamma(L+j_1)\Gamma(L+j_2)}{\Gamma(j_1+1)\Gamma(j_2+1)\Gamma(L-j_1)\Gamma(L-j_2)}\Bigg[\sum_{k=1}^{L+j_1-2}\Bigg(\binom{L+k+j_2-4}{k-1}\Big(2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-3,\\ 2L-j_1-j_2-1}} + \zeta_{\substack{L+j_1-k-1,\\ 3L-j_1+k-4}}\Big)\\
&-\binom{L+k+j_2-2}{k}\Big(2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-2,\\ 2L-j_1-j_2-2}} - 2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-1,\\ 2L-j_1-j_2-3}}\Big)\Bigg) + 2\zeta_{\substack{2L+j_1+j_2-3,\\ 2L-j_1-j_2-2}} + 2\zeta_{\substack{2L+j_2-1,\\ L+j_1-1,\\ 2L-j_1-j_2-3}}\ \zeta_{\substack{L+j_1-2,\\ 3L-j_1-3}} - \zeta_{\substack{2L+j_1+j_2-2,\\ 2L-j_1-j_2-3}}\Bigg]\\
&+\frac{\Gamma^2(2L-1)}{2\Gamma^2(L)}\Bigg[\sum_{k=1}^{2L-4}\Bigg(\binom{2L+k-4}{k}\Big(3\zeta_{\substack{2L-k-3,\\ 2L+k-2}} + 2\zeta_{\substack{2L-k-3,\\ 2L+k-4,\\ 2}}\Big) - 2\Big(\binom{2L+k-4}{k-1} + \binom{2L+k-4}{k+1}\Big)\zeta_{\substack{2L-k-3,\\ 2L+k-3,\\ 1}}\Bigg)\\
&+2\zeta_{\substack{2L-3,\\ 2L-4,\\ 2}} - 4(L-3)\zeta_{\substack{2L-3,\\ 2L-3,\\ 1}} + \zeta_{\substack{2L-4,\\ 2L-1}} - 4\zeta_{\substack{2L-2,\\ 2L-3}} + 4\zeta_{\substack{4L-6,\\ 1}} + 2\zeta_{\substack{3L-4,\\ 2L-2,\\ 1}} + \frac{2L+1}{L}\zeta_{2L-3}\zeta_{2L-2} - 5\zeta_{4L-5} - \zeta_{2L-3}^2\Bigg]\Bigg\}
\end{aligned}$$

Gurdogan, V.K. 2015

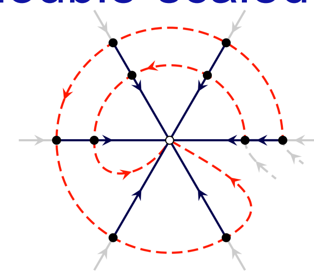
We extracted it from
Ahn, Bajnok, Bombardelli, Nepomechie 2013
and brought it
to explicit MZV form

Eden, Beisert, Staudacher, 2005

Dimensions from Asymptotic Bethe Ansatz

Caetano, Gurdogan, V.K 2016

- In bi-scalar model, rapidities live in “mirror” plane and the double-scaled ABA in SU(2) subsector (made of $\phi_1,\ \phi_2$)



$$(u_j^2+1/4)^L=\xi^{2L}\prod_{k\neq j}\frac{u_j-u_k-i}{u_j-u_k+i}\,\sigma^2(u_j,u_k)\,,$$

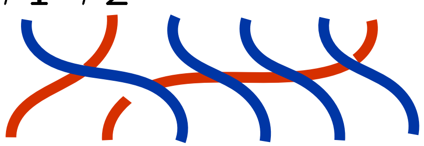
Where σ is mirror-mirror ABA dressing phase in g=0 limit

$$\sigma(u,v)=\frac{(1+4u^2)\,\Gamma\left(\frac{1}{2}-iu\right)\,\Gamma\left(\frac{3}{2}-iu\right)\,\Gamma(1+iu-iv)^2\,\Gamma\left(\frac{1}{2}+iv\right)\,\Gamma\left(\frac{3}{2}+iv\right)}{(1+4v^2)\,\Gamma\left(\frac{1}{2}-iv\right)\,\Gamma\left(\frac{3}{2}-iv\right)\,\Gamma(1+iv-iu)^2\,\Gamma\left(\frac{1}{2}+iu\right)\,\Gamma\left(\frac{3}{2}+iu\right)}\,.$$

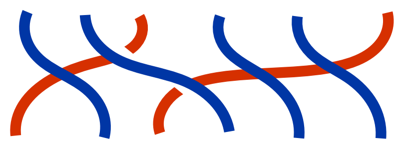
- The anomalous dimension are given by $\Delta=L-M+2i\sum_{j=1}^{M}u_j$

- Helps to compute unwrapped magnon graphs entering the mixing matrix:

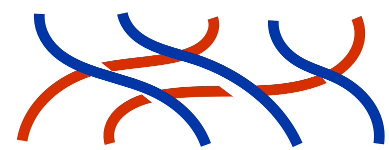
$\phi_1\ \phi_2$



I_a



I_b



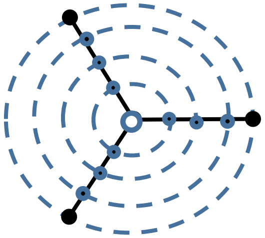
I_c

$$I_b\big|_{1/\epsilon}=-I_a\big|_{1/\epsilon}-\frac{160\zeta(3)}{9}+\frac{53\pi^4}{72}-\frac{187}{5}-\frac{25\pi^2}{12}$$

$$I_c\big|_{1/\epsilon}=I_a\big|_{1/\epsilon}+\frac{418\zeta(3)}{45}+\frac{121\pi^4}{360}+\frac{2\pi^2}{9}-\frac{112}{5}$$

- “Minimally connected” Feynman graph $I_a\big|_{1/\epsilon}$ is computed directly

Georgoudis, Goncalves, Pereira



Gromov, V.K., Korchemsky, Negro, Sizov 2017

Results for L=3 BMN vacuum

Gromov, V.K., Leurent, Volin 2013

- Constructing the double scaling limit of Quantum Spectral Curve (QSC) eqs. for twisted N=4 SYM we find the 2[nd] order Baxter equation for Q-functions

$$\left(\frac{(\Delta-1)(\Delta-3)}{4u^2}\pm\frac{i\,m}{u^3}-2\right)q(u)+q(u+i)+q(u-i)=0, \qquad (m=\xi^3)$$

where two solutions $q_2(u,m)$, $q_4(u,m)$ satisfy at u=0

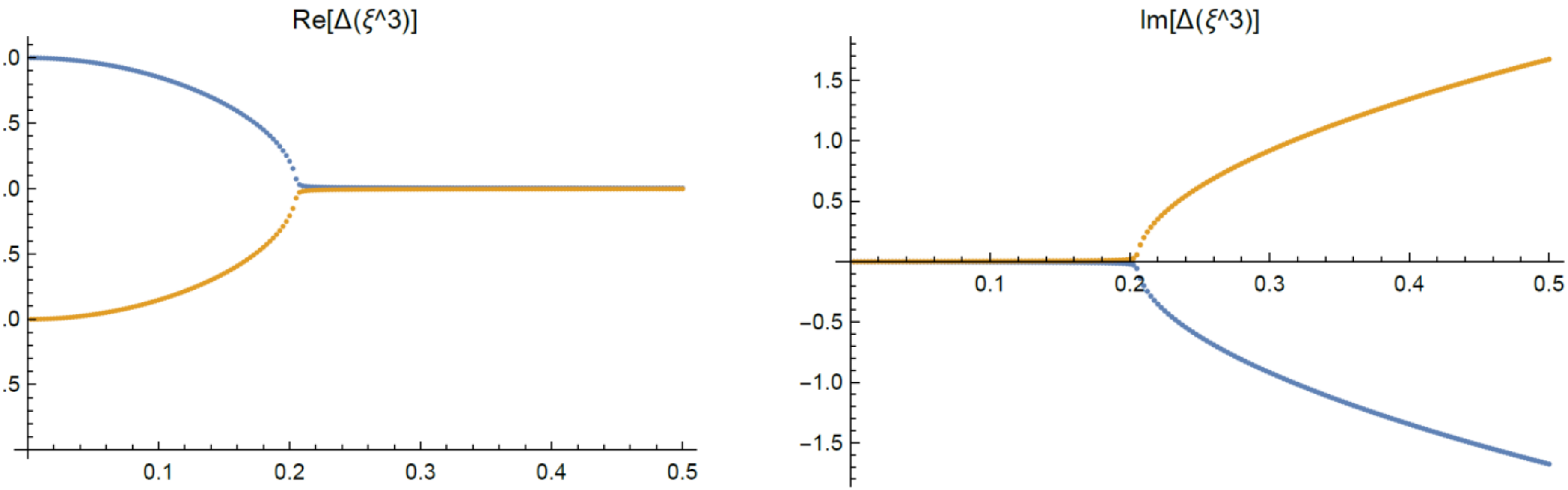
$$q_2(0,m)\,q_4(0,-m)\;+\;q_2(0,-m)\,q_4(0,m)=0$$

and have a "pure" large u asymptotics

$$q_2(u,m)\sim u^{\Delta/2-1/2}(1+\frac{\alpha_1}{u}+\frac{\alpha_2}{u^2}+\cdots), \qquad q_4(u,m)\sim u^{-\Delta/2+3/2}(1+\frac{\beta_1}{u}+\frac{\beta_2}{u^2}+\cdots)$$

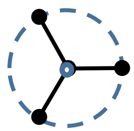
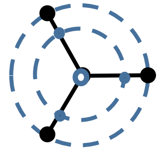
- We can study these equations perturbatively, iterating in ξ (wrappings) and reproducing systematically, order by order, the (periods of) wheel graphs
- We can also study the BMN dimension with great precision numerically, at all relevant couplings

Numerics for L=3 BMN vacuum

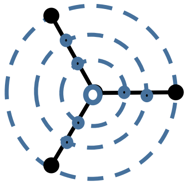
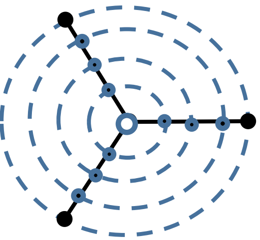


- Around ξ^3=0.2 dimension becomes imaginary: phase transition, finite convergency radius.
- What happens to the string dual?

L=3 BMN vacuum and all-loop wheel graphs

Gromov, V.K , Korchemsky, Negro, Sizov
(to appear)

$$\begin{aligned}
\Delta - 3 = \; & -12\xi^6\zeta(3) + \xi^{12}\left(189\zeta(7) - 144\zeta(3)^2\right) \\
& +\xi^{18}\left(-1944\zeta(8,2,1) - 3024\zeta(3)^3 - 3024\zeta(5)\zeta(3)^2 + \frac{198\pi^8\zeta(3)}{175} + 6804\zeta(7)\zeta(3) \right. \\
& \left. +\frac{612\pi^6\zeta(5)}{35} + 270\pi^4\zeta(7) + 5994\pi^2\zeta(9) - \frac{925911\zeta(11)}{8}\right) + \\
& \xi^{24}\left(\frac{10368}{5}\pi^4\zeta(8,2,1) + 5184\pi^2\zeta(9,3,1) + 51840\pi^2\zeta(10,2,1) - 148716\zeta(11,3,1)\right. \\
& -1061910\zeta(12,2,1) + 62208\zeta(10,2,1,1,1) - 93312\zeta(3)\zeta(8,2,1) - 288\zeta(3)^5 \\
& +72\gamma\pi^2\zeta(3)^4 - 77760\zeta(3)^4 - \frac{80756\pi^6\zeta(3)^3}{945} - 145152\zeta(5)\zeta(3)^3 - \frac{29}{270}\gamma\pi^8\zeta(3)^2 \\
& +\frac{9504\pi^8\zeta(3)^2}{175} - 879\pi^4\zeta(5)\zeta(3)^2 - 2025\pi^2\zeta(7)\zeta(3)^2 + 244944\zeta(7)\zeta(3)^2 \\
& +186588\zeta(9)\zeta(3)^2 + \frac{2910394\pi^{12}\zeta(3)}{2627625} - 2592\pi^2\zeta(5)^2\zeta(3) + \frac{29376}{35}\pi^6\zeta(5)\zeta(3) \\
& +12960\pi^4\zeta(7)\zeta(3) + 298404\zeta(5)\zeta(7)\zeta(3) + 287712\pi^2\zeta(9)\zeta(3) \\
& -5555466\zeta(11)\zeta(3) + 57672\zeta(5)^3 - 71442\zeta(7)^2 + \frac{13953\pi^{10}\zeta(5)}{1925} + \frac{7293\pi^8\zeta(7)}{175} - \frac{19959\pi^6\zeta(9}{5} \\
& \left. +\frac{119979\pi^4\zeta(11)}{2} + \frac{10738413\pi^2\zeta(13)}{2} - \frac{4607294013\zeta(15)}{80}\right) + O\left(\xi^{25}\right)
\end{aligned}$$

- Generalized to any number of spokes L is in work
  (Baxter equation is available)

Operators of length=3+2n and log-multiplets

Gromov, V.K , Negro, Korchemsky, Sizov (to appear this week)

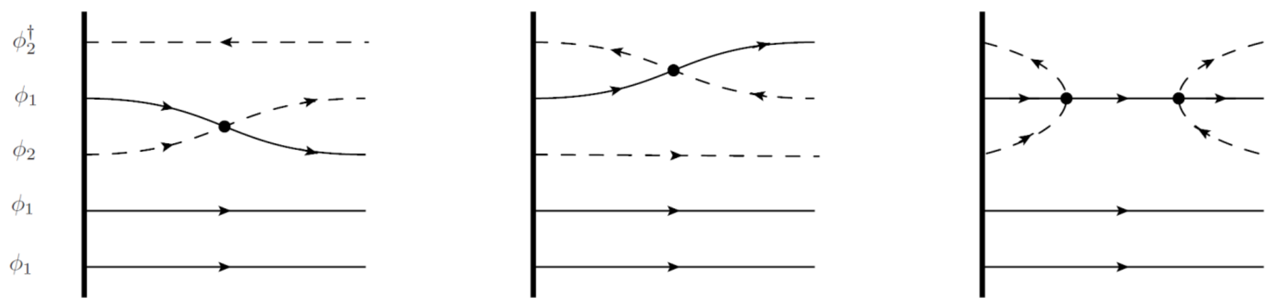
- Our Baxter eq. fixes only the charges of operator (0,Δ,0|J=3,0), not its length.
- It also describes operators of any length $L=3+2n_1+2n_2$ of the type

$$\mathcal{O} = \text{tr}\left(\phi_1^3 \ (\phi_1^\dagger \phi_1)^{n_1} \ (\phi_2^\dagger \phi_2)^{n_2}\right) + \text{permutations}$$

- Protected operators: $\mathcal{O} = \text{tr}\left(\phi_1^J \ (\phi_1^\dagger \phi_1)^{n_1}\right) + \text{permutations}$
- Mixing in multiplet of operators of length L=5:

protected

$$O_1 = \text{tr}\,(\phi_1^3\phi_2\phi_2^\dagger)\,, \qquad O_2 = \text{tr}\,(\phi_1^2\phi_2\phi_1\phi_2^\dagger)\,, \qquad O_3 = \text{tr}\,(\phi_1\phi_2\phi_1^2\phi_2^\dagger)\,, \qquad O_4 = \text{tr}\,(\phi_2\phi_1^3\phi_2^\dagger)\,.$$



- Non-unitary mixing matrix. “Diagonalisation” means bringing to Jordan form

$$\mu\frac{d}{d\mu}O_i(x) = V_{ij}O_j(x), \qquad V = \frac{1}{4\pi^2}\begin{bmatrix} 0 & \xi^2 & 0 & 0 \\ 0 & 0 & \xi^2 & 0 \\ 0 & -\xi^4 & 0 & \xi^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U\cdot\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -i\xi^3 & 0 \\ 0 & 0 & 0 & i\xi^3 \end{pmatrix}\cdot U^{-1}$$

- Lower block contains usual operators with dimensions $\Delta_{\pm}^{L=5} = 5 \pm 2i\left(\frac{\xi}{4\pi}\right)^3 + \ldots$
- Upper block is Jordan cell, it gives log-conformal correlators:

$$\langle \tilde{\mathcal{O}}_\alpha^\dagger(x)\tilde{\mathcal{O}}_\beta(0)\rangle_{\text{ren}} = \frac{1}{x^{10}}\begin{bmatrix} 1 & \frac{1}{2}\log x^2 \\ 0 & \frac{1}{4}\log^2 x^2 \end{bmatrix}_{\alpha\beta}$$

Caetano 2016

Perturbative expansion and numerics for operators of length L>3

- At L=3+4n+2 Baxter equation renders two complex conjugate dimensions

$$\Delta_-^{L=5} = 5 - 2i\xi^3 + 3\xi^6 + \frac{31}{4}i\xi^9 + \left(3\zeta_3 - \frac{97}{4}\right)\xi^{12} + i\left(\frac{27\zeta_3}{2} - \frac{5359}{64}\right)\xi^{15} + \left(-\frac{219\zeta_3}{4} - \frac{15\zeta_5}{2} + \frac{4911}{16}\right)\xi^{18} + \dots$$

$$\Delta_+^{L=5} = (\Delta_-^{L=5})^*$$

$$\Delta_-^{L=9} = 9 - \frac{i\xi^3}{3} + \frac{7\xi^6}{216} - \frac{223i\xi^9}{10368} + \xi^{12}\left(\frac{\zeta_3}{432} + \frac{17029}{1119744}\right) + \xi^{15}\left(\frac{1424867i}{214990848} - \frac{31i\zeta_3}{31104}\right) + \dots$$

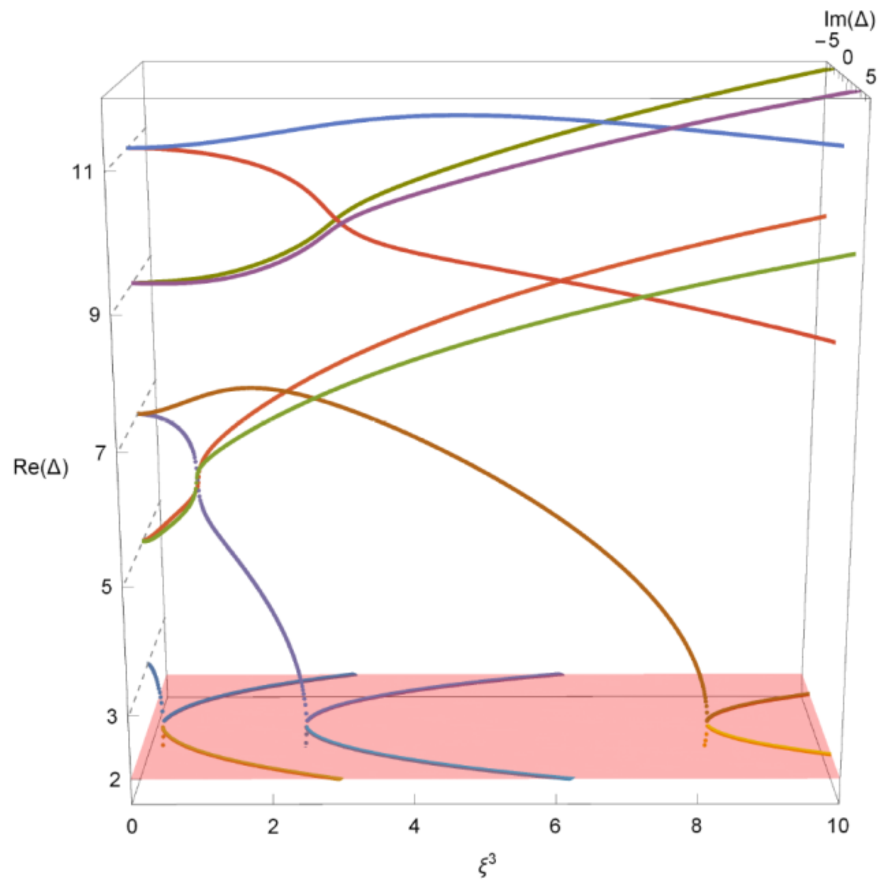
$$\Delta_+^{L=9} = (\Delta_-^{L=9})^*$$

- At L=3+4n dimensions are real (until tl

$$\Delta_{7,A} = 7 - \frac{\xi^6}{2} - \frac{17\xi^{12}}{64} - \frac{891\xi^{18}}{4096} - \frac{27465\xi^{24}}{131072} + \dots$$

$$\Delta_{7,B} = 7 + \frac{\xi^6}{2} - \frac{23\xi^{12}}{64} + \xi^{18}\left(\frac{15283}{36864} - \frac{\zeta_3}{12}\right) + \xi^{24}\Big($$

- Numerics:



Asymptotics of large ξ and quasiclassics at large L,ξ

- These asymptotics come from classical, finite gap type solution of Baxter
- Strong coupling $\xi \to \infty$ at finite length: $L = 3 + 2N \qquad (N - \text{even})$

$$-(\Delta-3)(\Delta-1) = \frac{2\xi^3}{N} + N^2 + \frac{N^3\left(3N^2+4\right)}{16\xi^3} + \frac{N^4\left(9N^4+56N^2+16\right)}{64\xi^6} +$$
$$+\frac{N^5\left(153N^6+2200N^4+3728N^2+512\right)}{1024\xi^9} + \frac{N^6\left(195N^8+5096N^6+22176N^4+19584N^2+1792\right)}{1024\xi^{11}} + O\left(\frac{1}{\xi^{15}}\right)$$

- Classical limit: $\Delta \sim L \sim u \to \infty, \qquad x \equiv u/\xi\,, \qquad \mathcal{D} \equiv \frac{\Delta}{8\xi}\,, \qquad \mathcal{L} \equiv \frac{N}{2\xi} \to \text{fixed}$

- Baxter eq. becomes finite gap eq. for a classical non-compact 3-spin system

$$\cos(p(x)) = +\frac{i}{2x^3} - \frac{8\mathcal{D}^2}{x^2} + 1\,, \qquad \text{where} \quad p(x) \equiv \frac{1}{i}\log\frac{q(u+i/2)}{q(u-i/2)} \simeq \partial_u \log q(u)$$

- Exact quantisation condition replaced by standard Bohr-Sommerfeld relation!

$$\frac{1}{2\pi i}\oint_\gamma p(x)dx = \mathcal{L} + \frac{1}{2\xi}$$

- Choosing appropriately the contour one finds explicit answer:

$$\mathcal{L} = -\frac{1}{16\mathcal{D}^2}\,{}_3F_2\left(\frac{1}{3},\frac{1}{2},\frac{2}{3};1,\frac{3}{2};-\frac{27}{4096\mathcal{D}^6}\right)$$

- At small $\mathcal{L}$

$$\mathcal{D} = \pm i\frac{1}{4\sqrt{\mathcal{L}}}\left(1 + \mathcal{L}^3 + \frac{5\mathcal{L}^6}{2} + \frac{31\mathcal{L}^9}{2} + \frac{1075\mathcal{L}^{12}}{8} + \frac{11095\mathcal{L}^{15}}{8} + \ldots\right)$$

Caetano, Gurdogan, V.K.

Strongly twisted ABJM in double scaling limit

- The ABJM, ‘t Hooft coupling is defined through the CS level $\lambda = \frac{N_c}{k}$ and there are also 3 twists $q_i \equiv e^{-i\gamma_i}$ corresponding to SU(4) → $U(1)^3$
- Similar double scaling limit, weak coupling corresponds to large CS level

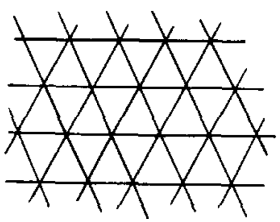
$$q_1, q_2, q_3 \to \infty, \qquad \lambda \to 0$$

where as $\xi_1 \equiv q_1 \lambda^{2/3}, \quad \xi_2 \equiv q_2 \lambda^{2/3}, \quad \xi_3 \equiv q_3 \lambda^{2/3} \quad - \quad \text{fixed}$

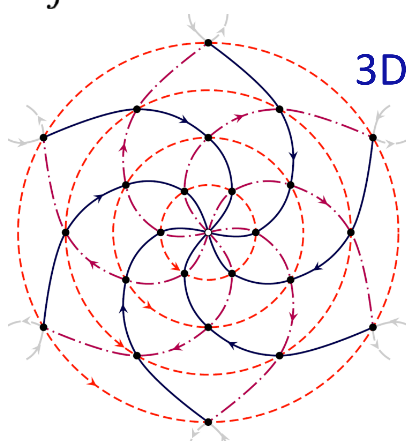
- Gauge fields, some 6-scalar and all “Yukawa” interactions decouple, ABJM becomes a chiral 3D QFT of 3 interacting scalars in 3 dimensions

$$\mathcal{L}^{(DS)}_{\text{ABJM}} = \text{Tr}\left[-\partial^\mu Y^\dagger_j \partial_\mu Y^j + \quad \xi^3 Y^1 Y^\dagger_2 Y^3 Y^\dagger_1 Y^2 Y^\dagger_3\right] \qquad \xi^3 \equiv \xi_1 \xi_2 \xi_3$$

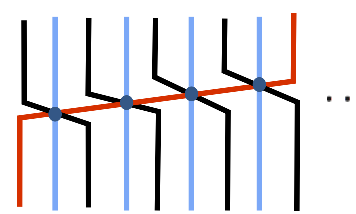
triangular fishnet graphs



3D wheel



single magnon

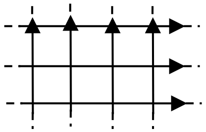


Chicherin, V.K., Loebbert, Mueller, Zhong

- Yangian invariant amplitudes: fishnet graphs cut out of triangular lattice

Conclusions and prospects

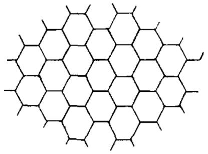
- Double scaling limit of twisted N=4 SYM theory produces new 4-dimensional chiral CFTs explicitly integrable in planar limit at each loop order
- Physical quantities are dominated at each loop order by a single "fishnet" Feynman graph with specific boundary – integrable SU(2,2) spin chain
- Possible to compute multi-loop graphs in 4D and 3D with fishnet structure



- Bi-scalar amplitudes are finite and exhibit explicit Yangian invariance
- Structure constants? 1/N corrections? β-functions (study of scale dependence)? quark-antiquark potential ……. in DS limit
- We computed from QSC exactly J=3 BMN dimensions and corresponding wheel graphs. How to compute them from conformal spin chain? Generalization to any J?

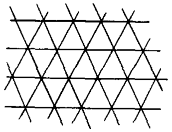
Gromov, Levkovich-Maslyuk 2016

- Similar observation for DS limit of ɣ-deformed 3D ABJM (no conformal anomaly)

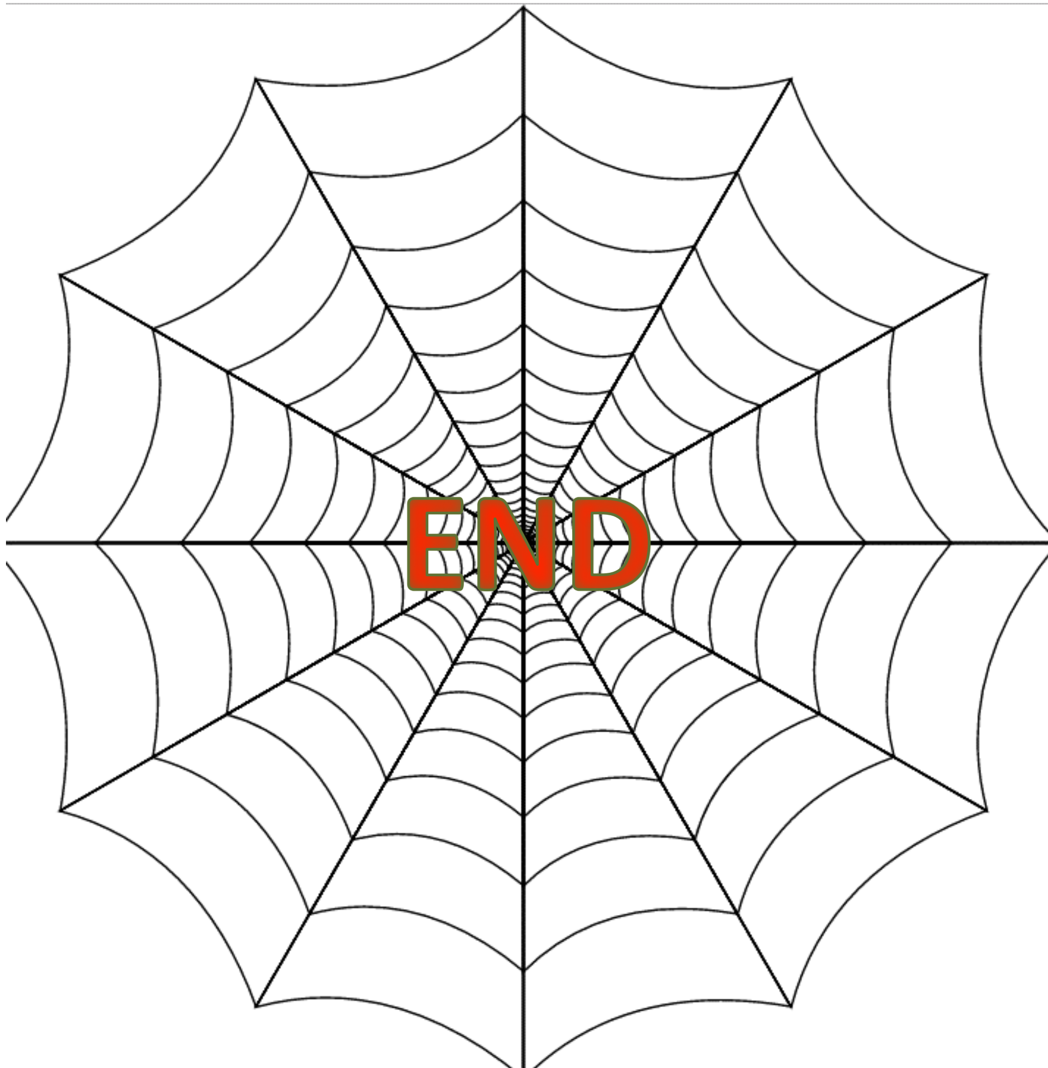


Torrens, Mamroud 2017

- 6D tri-scalar theory: regular hexagonal graphs. Twisted 6D SYM?



- Our observations shed some light on the origins of integrability of planar SYM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